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RESEARCH ARTICLE

Characteristics of the Dynamic Strategic Civilizational System $\dot{X}$=Ax+g

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ABSTRACT

The aim of this paper is to introduce some new applications to explain the behaviors of dynamic phenomena of civilized strategies, and how to determine the basic behavior of the system in area of cybernetics gaming {17}i.e. a field of total perceived power differential games theory and national strategy system, specially the control, stability and Observability.

INTRODUCTION

The aim of this paper is to introduce some new applications to explain the behaviors of dynamic phenomena of civilized strategies, and how to determine the basic behavior of the system in area of cybernetics gaming {17}i.e. a field of total perceived power differential games theory and national strategy system, specially the control, stability and Observability.

$$(1) X = \dot{A}x + g$$

$$A = (K_{ij}), g = \begin{bmatrix} g_1 \\ g_2 \\ . \\ . \\ . \\ g_n \end{bmatrix}$$

And it is equivalent to the equation $\dot{X} = Ax + Bu, \quad x(0) = x_0$ (2)

Then $u \in L_\infty(0, t, IR)$ and $A \in \Psi(IR''), B \in (IR''', IR'')$ or $L_2(0, t, IR''')$

And is open to choice and is called control and thus $x \in L_\infty(0, t, IR)$ therefore absolutely continuous and achieves the equation (2)

$$x(t) = e^{At}x_0 + \int_0^t e^{A(t-s)}Bu(s)ds \quad (3)$$

Theorem (1.1): The control space X is a subspace in IR''

Proof:

Let $x_1, x_2 \in \Psi$ then $\exists\, t_1, t_2 < \infty$ and controls $u_1 \in L_\infty(0, t_1, IR'')$, $u_2 \in L_\infty(0, t_2, IR'')$ such that

$$x_1 = \int_0^{t_1} e^{A(t_1-s)} Bu_1(s)ds$$

$$x_2 = \int_0^{t_2} e^{A(t_2-s)} Bu_2(s)ds$$

Now from u_1 generated control $\hat{u}_1 \in L_\infty(0, t_2, IR'')$

$\hat{u} = 0$, if $0 \leq t \leq t_2 - t_1$

$\hat{u}(t + t_2 - t_1) = u(t), \quad 0 \leq t \leq t_1$

Now the control

$\alpha\hat{u}_1(t) + \beta u_2(t) \in L_\infty \; such \; that - \infty < \alpha, \beta < \infty$

$x = \alpha \int_{t_1}^{t_2} e^{A(t_2-s)} Bu(s - t_2, t_1)ds + \beta x_2$

Now we set

$$x = \alpha \int_0^{t_2} e^{A(t_1-s)} Bu_1\,(s')ds' + \beta x_2 = \alpha x_1 + \beta x_2$$

Hence if $x_1, x_2 \in X \; then \; \alpha x_1 + \beta x_2 \; also \; belongs \; to \; X \;\; \forall \;\; \alpha, \beta \; finite$

for that X is linear subspace in IR″

Preface (1.1): the Vector x is Perpendicular to Space X if and only if

$$\begin{bmatrix} B' \\ B'A' \\ . \\ . \\ . \\ B'(A')^{n-1} \end{bmatrix} x = 0$$

Theorem (1.2): The dimension of the control space is the rank of the matrix $[B, AB, \ldots A^{n-1}B]$ the rank of this matrix =n∗m and is called control matrix (18)

Proof:

If x is Perpendicular on x so since $x_1 = \int_0^{t_1} e^{(t_1-s)}\, Bu(s)ds$ is an element identical to elements of x then we find

$$\langle x, \int_0^{t_1} e^{(t_1-s)}\, Bu(s)ds \rangle = 0 \qquad (4)$$

Where $\langle , \rangle$ symbolized inner product in x

But the equation (4) includes

$$\int_0^{t_1} \langle B' e^{A(t_1-s)} x, u(s)ds \rangle = 0 \quad \forall \; controls \; u_1 \; and \; then \; B' e^{A(t_1-s)} x = 0$$

$\forall \; 0 \leq s \leq t_1 \; put \; s = t_1 \; we \; find \; B'x = 0$

and by differention with respect to the variable s "k" times and let s = t_1

$B'(A')^k x = 0$, $k = 1,2,3, \ldots, (n-1)$ Conversely assume Preface (1.1) is achieve by using Caily – Hamilton theorem there is exit $C_1, C_2, \ldots, C_n$ so that $(A')'' = C_1(A')^{n-1} \ldots + C_n I$

So that $B'(A')^n x = 0$, and thus, we can continue to obtain $B'(A')^k x = 0 \; if \; 0 \leq k \leq \infty$

Now

$$B'e^{A(t_1-s)}x = B'\left[I + (t_1 - s)A' + \frac{(t_1 - s)}{2!}\right](A')^2 + \cdots$$

Therefore $B'e^{A(t_1-s)}x = 0$ Thus, this indicates that

$$\int_0^{t_2} \langle x, B'e^{A(t_1-s)}Bu(s)\rangle\, ds = 0 \;\; \forall\, u \text{ and } t_1 \text{ therefore } X \text{ it should be vertical on } x$$

Now

$$rank\,[B, \quad AB, \ldots \quad A^{n-1}] = rank \begin{bmatrix} B' \\ B'A' \\ . \\ . \\ B'(A')^{n-1} \end{bmatrix} x = 0$$ It n-1 the number of independent linear X such that:

$$\begin{bmatrix} B' \\ B'A' \\ . \\ . \\ B'(A')^{n-1} \end{bmatrix} = 0$$ From preface (1.1) and so after x it is n the number of independent linear X such that X $vertical\ on\ x$ then after $\dim x = rank\,[B, \quad AB, \ldots \quad A^{n-1}]$

Preface (1.2): if it was $rank\,[B, \quad AB, \ldots \quad A^{n-1}] = n \text{ then all points in }$ IR″

It can be accessed from the origin point, so controlling the strategic system from the origin point can be verified in an easy algebraic way.

Theorem (1.3): if it was $rank\,[B, \quad AB, \ldots \quad A^{n-1}] = n\,, So\ \forall\ t_1 \geq 0 \text{ the matriex}$

$W(t_1) = \int_0^t e^{-At}BB'e^{-At}dt$ that is strictly positive because any finite linear operator N on a Hilbert space H, is regular if NN*=N*N addition, there is a control that leads any

$x_0 \in IR''$ To the origin.

Proof: if it was $[B, \quad AB, \ldots \quad A^{n-1}] = n$, so from the proof of theorem (1.2) we see that $B'e^{-At}x = 0 \quad 0 \leq t \leq t_1 \text{ it show that } X = 0$

$$\langle x, W(t_1)x\rangle = \int_0^t \langle B'e^{-At}x, B'e^{-At}x\rangle dt = 0 \int_0^{t_1} \|B'e^{-At}x\|^2 dt \;\; \text{hence } \boldsymbol{W(t_1)}$$

At least completely positive but $\langle x, W(t_1)x\rangle$ indicate that $B'e^{-At}x = 0$ this achieves that X=0 therefore $\boldsymbol{W(t_1)}$ clearly absolutely positive now consider that:

$u(t) = B'e^{-Au}(W(t))^{-1}x_0$ Control is played over the interval $(0, t_1]$ is for a system with an initial state x_0, then $x(t_1) = e^{-At}x_0 - \int_0^t e^{A(t_1-t)}BB'e^{-As}(W(t_1))^{-1}x_0 x dx$

$$x(t_1) = e^{-At}x_0 - e^{A(t_1)}\,W(t_1)\,(W(t_1))^{-1}x_0$$

This control leads to the origin so, Synthesis of theories (1.1) and (1.2) results:

Theorem (1.4): System (2) is controllable if and only if $rank\,[B, \quad AB, \ldots \quad A^{n-1}] = n$

Now that the concept of control in the strategic context of civilization is fundamental, geometrically we expect that control is stable according to the changes in the coordinates. To see the truth of this, let y = PX where P the matrix is non-existent, then the equation (2) becomes in the form:

$\dot{y} = \hat{A}y + \hat{B}u$ So that $\hat{A} = PAP^{-1} \quad , \hat{B} = PB$

$$\begin{aligned} rank\,\left|\hat{B}, \hat{A}\hat{B}, \ldots, \hat{A}\hat{B}^{n-1}\right| &= rank|PB, PAP^{-1}PB, \ldots, PA^{n-1}B| = rank\ P|B, AB, \ldots, A^{N-1}B| \\ &= rank\ |B, AB, \ldots, A^{N-1}B| \end{aligned}$$

Example (1.1): Consider that the standard differential equation of the strategic dynamic civilizational race system between nations is:

$$\frac{d^n x}{dt^n} + a_n \frac{d^{n-1} x}{dt^{n-1}} + \cdots + a_1 x = u \;\; let\; X = x_1 \qquad (5)$$

$\frac{dx_1}{dt_1} = x_2, \frac{dx_2}{dt_2} = x_3, \ldots, \frac{dx_{n-1}}{dt_{n-1}} = x_n$ Then we find:

$\dot{X} = \dot{x} + g = Ax + bu$ So that

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ x_n \end{bmatrix}, b = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \\ 1 \end{bmatrix}, A = \begin{bmatrix} 010 & \ldots & \ldots & 0 \\ 001 & \ldots & \ldots & 0 \\ 0 & \ldots & \ldots & 00 \\ \vdots & \vdots & \vdots & \vdots \\ -a_1 & \ldots & \ldots & a_n \end{bmatrix}$$

Then the rank$[A, Ab, \ldots, A^{n-1}b] = n$

System (5) is controllable. It is clearly a competitive system for a (n) of nations in the strategic civilizational race.

Example (1.2): This example shows that not every strategic system is controllable. Consider that

$$\begin{bmatrix} \bullet \\ x \\ \bullet \\ y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \;\; so\; [b, Ab] = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \; and\; rank \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} = 1$$

So the system is not controllable, and this is obvious if we write the system explicitly as follows:

$$\overset{\bullet}{y} = x \quad , \underset{\bullet}{y} = y + u$$

It is clear that X(t) that it is completely determined by the initial position and is not affected by the control u.

2/ Stability: If the controls for the system are given as a function of time I mean $u: [0, T] \to IR'''$then we call them an open loop. The reason is that if an open loop control is designed it must be based entirely on the mathematical model of the system (1) or (2). Thus if the actual dynamics differ from that or the control model, the open loop control is likely to be ineffective. However, if loop control is used, although this fits the model at least it takes into account the continuity of the actual state of the system. To develop the control, the real issue is to design controls so that the equilibrium point of an unstable system can be made relatively stable ([1], [4], [13], [14], [15], [16], [20]).

Definition of Stability: We say that the origin of a system (2) is stable if there exists $aD \in \varepsilon(IR''', IR'')$ such that the system $\dot{X} = Ax + BDx$ has a comparatively stable origin.

Theorem (2.1): if the system (2) is controllable then it is stable.

Proof: since the system (2) is controllable by using theorem (1.3) we find:

$W(\varepsilon) = \int_0^t e^{At} BB' e^{At} dt, \quad \varepsilon > 0$ It is exactly positive and therefore exists K> 0 such that

$$W(\varepsilon) \geq KI$$

Now considering the Lipunov function [1] and equation $V = \langle x, P^1 x \rangle$ such that

$P = \int_0^v (v - t) e^{-At} BB' e^{-At} dt, \quad for\; v > \varepsilon$ So

$$P > \int_0^v (v - t) e^{-At} BB' e^{-At} dt = (v - \varepsilon) W(\varepsilon) \geq (v - \varepsilon) KI$$

Thus $\boldsymbol{P^{-1}}$ is defined complete now

$\dot{v} = \langle x, (A'P^{-1} + P^{-1}A)x\rangle + \langle zx, P^{-1}Bu\rangle + \langle bu, P^{-1}x\rangle =$

$\langle y, (PA' + AP)y\rangle + \langle y, Bu\rangle + \langle Bu, y\rangle$ Such that $y = P^{-1}x$ but

$PA' + AP = \int_0^v (v-t)[e^{-At}BB'e^{-At}A' + e^{-At}BB'e^{-At}]dt = -\int_0^v (v-t)\frac{d}{dt}[e^{-At}BB'e^{-At}]dt =$
$-[(v-t)\, e^{-At}BB'e^{-At}]_0^v - \int_0^v e^{-At}BB'e^{-At}dt = vBB' - \int_0^v e^{-At}BB'e^{-At}dt$ Thus we find that

$$\dot{V} = \|B'y\|^2 + \langle y, Bu\rangle + \langle Bu, y\rangle - \langle y, \int_0^v e^{-At}BB'e^{-At}ydt\rangle$$

Choose:

$u = -vB'y$

Such that

$Dx = B'P^{-1}x$

So

$\dot{V} \le -v\|B'y\|^2 - \langle y, W(\varepsilon)y\rangle \, and$

$\dot{V} \le -v\|B'y\|^2 - \langle x, P^{-1}W(\varepsilon)P^{-1}x\rangle$ Now that $P^{-1} and\, W(\varepsilon)$ clearly positive, so the origin point is stable and convergent [12].

3/ Observation:

For system (2) we have no further status or condition X so that the most important consideration is whether it is possible to recreate from the observations the state of the system. To pose this as a mathematical problem, let us assume that the system (2) is expanded by the equation

$$Y = CX \qquad (6)$$

Such that $C \in \xi(\mathrm{IR}'', IR^k)\, , k \le n$ suppose we can observe Y that given the equation (3)

$x(t) = e^{At}x_0 + \int_0^t e^{A(t-s)}Bu(s)ds$ We have

$y(t) = Ce^{At}x_0 + \int_0^t Ce^{A(t-s)}Bu(s)ds$ if we put

$\bar{y}\,(t) = y(t) - \int_0^t Ce^{A(t-s)}Bu(s)ds$ So we get:

$\bar{y}(t) = Ce^{A(t-s)}$ And observe that $y \in C^{\infty}\lfloor 0, \infty, IR^k\rfloor$

Now if we assume that u and y are given information at some interval$(0, t_1]$, we can ask whether this information is important for determining x_0. This leads us to the following definition:

Definition (3.1) observable Systems:

We say that the extended system (2) with the equation (6) is Monitoring and observable$(0, t_1]$, if it is given $u \in L_{\infty}\lfloor 0, t_1, IR^k\rfloor$ given y as an absolutely continuous function on$(0, t_1]$. Then it is possible to determine x_0 a single property of the equation (7). This is equivalent to the problem of determining x_0 from $\bar{y} \in C^{\infty}[0, t_1, IR^k]$.

Theorem (3.1): the strategic system governed by the two equations (2) and (6) is observable if and only if $rank\, \lfloor C'', A', C'', \dots, (A')^{n-1}C''\rfloor = n$

Proof: It is clear that knowing $\bar{y} \in (0, t_1)$ leads to a unique value for the variable x_0 if and only if $Ce^{At}x_0 = 0 \quad for\ 0 \le t \le t_1\ then\ x_0 = 0$ put we seen in the proof of preface (1.1) this condition is equivalent to the following:

$$rank\begin{bmatrix} C \\ CA \\ . \\ . \\ CA^{n=1} \end{bmatrix} = n \; or \; that \; rank \; \lfloor C'', A', C'', \dots, (A')^{n-1}C'' \rfloor = n \; We \; note \; that \; since$$

$\bar{y} \in C^{\infty}[0, t_1, IR^k]$ *Then we have* $\bar{y}\ (0) = Cx_0$, $\bar{y}(0) = CAx_0$, *now* $\bar{y}$ *and* x_0

Related by these equations, then $rank\ \lfloor C'', A', C'', \dots, (A')^{n-1}C'' \rfloor = n$.

Then they are consistent and have a unique solution x_0 in our model (2). The observations are likely to be disturbed by the turbulence, in which case the above analysis is necessary but not sufficient.

Definition (3.2) Binary:

We say that the Civilizational strategic system

$$S \left\{ \begin{matrix} \dot{X} = Ax + Bu \\ Y = Cx \end{matrix} \right\}$$

$$S^* \left\{ \begin{matrix} \dot{X} = \tilde{A}x + \tilde{B}u \\ Y = \tilde{C}x \end{matrix} \right\}$$

They are binaries if $\tilde{A} = A'$, $\tilde{B} = C''$, $\tilde{C} = B'$

Theorem (3.2): if the system S is observable then the rank of matrix $[C'', A', C'', \dots, (A')^{n-1}C''] = n$ so $rank\ \lfloor \tilde{B}, \tilde{A}\tilde{B}C'', \dots, (\tilde{A})^{n-1}\tilde{B} \rfloor = n$ which indicates that S^* it is observable.

Example (3.1): Consider the standard differential equation for the strategic system as:

$$\frac{d^n x}{dt^n} + a_n \frac{d^{n-1}x}{dt^{n-1}} + \dots + a_1 x = u \ \ , y = x$$

Then it is possible to convert this into a matrix differential equation in the same manner as the example (1.1) to obtain.

$$\dot{X} = Ax + Bu$$

$$y = C'x \; , such \; that \; C^1 = (1,0, \dots ,0); \; b^1 = (0, \dots ,1)$$

$$A = \begin{bmatrix} 010 & \dots & \dots & 0 \\ 001 & \dots & \dots & 0 \\ 0 & \dots & \dots & 00 \\ \vdots & \vdots & \vdots & \vdots \\ -a_1 & \dots & \dots & a_n \end{bmatrix}$$

Then $rank\ \lfloor C, A', \dots, (A')^{n-1}C \rfloor = n$ from theorem (3.2) $rak \left[C, \tilde{A}, \dots, \left(\tilde{A}\right)^{n-1} C \right] = n$

Therefore, the system is observable.

CONCLUSION:

The strategic dynamic civilization race system between nations (2) is controllable if the control space dimension equals n, and consequently, it is either stable or can be stabilized. Therefore, to understand the behavior of system (2), it is necessary to study the controllability and stability of the strategic civilization system, i.e., to design controls such that the equilibrium point of the unstable system can be made asymptotically stable.We conclude that all points in the space can be reached from the origin. Additionally, we deduce that not every system is controllable, and that the two strategic systems S and S∗ are observable if they are duals of each other. Accordingly, system (2) and its expanded version defined by the equation y=Cx are observable if the rank of the controllability matrix equals the control space dimension.

Thus, the functional analysis approach is highly significant and very useful in analyzing and understanding the characteristics of the strategic civilization system (2).

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