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RESEARCH ARTICLE

The Influence of Gravity on Relativistic

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ABSTRACT

This study examines the influence of gravitational force on the relativistic formulations of mass, length, and time in the context of weak gravity. This effect is characterised by the dependence of these expressions on the temporal component of the metric tonnes inside the weak field approximation. Consequently, it is believed that incorporating spatial components of this tensor or higher derivatives of this approximation may facilitate a further generalisation of these expressions to accommodate the equivalent higher-order effects of a gravitational field.

INTRODUCTION

The theory of Special Relativity (SR) is termed as such because it can be considered a specific instance of General Relativity (GR) in the absence of gravitational forces, whereas the latter encompasses the broader framework that includes the influence of gravity. In summary, General Relativity addresses the motion of matter in relation to non-inertial accelerated reference frames, whereas Special Relativity exclusively focusses on linear motion inside inertial frames. In geometric terms, this indicates that General Relativity (GR) delineates the motion of matter inside a curved spatial framework, whereas Special Relativity (SR) pertains to flat spatial geometry. This notion embodies the Equivalence Principle [1]. This concept asserts that the principles governing accelerated motion inside an infinitesimally small region of curved space, under the influence of gravitation, resemble those of unaccelerated motion in the absence of a gravitational field.

Conversely, historically, both General Relativity and Special Relativity were driven by distinct concepts. This condition renders general relativity exceedingly isolated from the mainstream physical laws, to the degree that special relativity fails to establish a common foundation for reconciling general relativistic principles with those of other branches of physics. The special relativistic framework of physics is apparent in both classical and quantum theories.

This debate stems from viewing special relativity as a particular instance of general relativity, which is a misleading notion. It appears logical to consider that Special Relativity serves primarily as a mathematical framework for articulating physical principles; furthermore, this theory, without expressions that denote the impact of force fields, provides an inaccurate depiction of nature. Consequently, to mitigate this deficiency, we examine the impact of including gravitational potential via metric tensor components on the special relativistic formulations of time, length, and mass. [2]

Special relativity in the presence of gravitation:

In RS the time, mass, and length can be respectively obtained [3] in a moving frame by either dividing or multiplying their values in the rest frame by factor γ as the following:

$$m = \frac{m_0}{\gamma} dt, dt = \frac{dt_0}{\gamma}, and\ dl = \gamma dl_0 \quad (1)$$

Where the subscript 0 stands for the quantity measured in rest frame and,

$$\gamma = \sqrt{1 - \frac{v^2}{c^2}} \quad (2)$$

To see how gravity affects these quantities it is convenient to express γ in terms of the proper time in the usual way [4]

$$c^2 d\tau^2 = c^2 dt^2 - d\chi^i d\chi^i, \chi^0 = ct \quad (3)$$

i.e.,

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{1}{c^2}\frac{d\chi^i}{dt}\frac{d\chi^i}{dt}} = \sqrt{1 - \frac{v^2}{c^2}} = \gamma \qquad i = 1,2,3 \quad (4)$$

Thus, we can easily generalize γ to include the effect of gravitation by using (3) and by adopting the weak field approximation [6] in terms of the potential ⍰ were,

$$g_{11} = g_{22} = g_{33} = -1, g_{00} = 1 + \frac{2⍰}{c^2} \quad (5)$$

Which yields

$$\gamma_g = \frac{d\tau}{dt} = \sqrt{g_{00} - \frac{1}{c^2}\frac{d\chi^i}{dt}\frac{d\chi^i}{dt}} = \sqrt{g_{00} - \frac{v^2}{c^2}} \quad (6)$$

This means that when the effect of motion only is considered, as in SR the expression for γ_g coincides with (2) and in view of (1) dt takes the form

$$dt = \frac{dt_0}{\sqrt{1-\frac{v^2}{c^2}}}, \quad (7)$$

While if the effect of gravity only is considered this expression is given by:

$$dt = \frac{dt_0}{\sqrt{g_{00}}} \quad (8)$$

In view of equations (7), (8) and (6) the expression

$$dt = \frac{dt_0}{\gamma}, \quad (9)$$

Can be generalized so as to recognize the effect of motion as well as gravity on time, to get

$$dt = \frac{dt_0}{\sqrt{g_{00}-\frac{v^2}{c^2}}}, \quad (10)$$

A similar result can be obtained for the length and hence for the volume where the effect of motion and gravity alone, respectively reads,

$$V = V_0\sqrt{1 - \frac{v^2}{c^2}} \quad (11)$$

And

$$V = V_0\sqrt{g_{00}} \quad (12)$$

The generalized expression con be obtained in view of (4) by,

$$V = V_0\gamma_g = V_0\sqrt{g_{00} - \frac{v^2}{c^2}} \quad (13)$$

Additionally, to generalise the concept of mass to encompass the influence of gravitation, we utilise the expression for the Hamiltonian density as presented in General Relativity [7], and using (6) we derive:

$$H = \rho c^2 = g_{00}T^{00} = g_{00}\rho_0(\frac{d\chi^0}{d\tau})^2 = g_{00}\frac{\rho_0 c^2}{\gamma_g^2} = g_{00}\frac{m_0 c^2}{V_0\gamma_g^2} \quad (14)$$

These yields:

$$\rho c^2 = \frac{mc^2}{V} = \frac{g_{00} m_0 c^2}{\gamma_g V} \qquad (15)$$

Therefore

$$m = \frac{g_{00} m_0}{\sqrt{g_{00} - \frac{v^2}{c^2}}} \qquad (16)$$

Which is the expression of mass in the presence of gravitation. Using equation (5) and (16) when the field is weak and the speed is small, the energy E is given by:

$$E = mc^2 = \frac{g_{00} m_0 c^2}{\gamma_g} = m_0 g_{00} \left(g_{00} - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} c^2 \qquad (17)$$

$$= m_0 \left(1 + \frac{2\square}{c^2}\right)\left(1 + \frac{2\square}{c^2} - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} c^2$$

$$\approx m_0 \left(1 + \frac{\square}{c^2} + \frac{1}{2}\frac{v^2}{c^2}\right) c^2$$

$$E = m_0 c^2 + \frac{1}{2} m_0 v^2 + m_0 \square$$

$$E = m_0 c^2 + K + U \qquad (18)$$

Where K is the kinetic energy and U the potential energy. Equation (17) shows that the energy in SR reduces to an expression not including the potential energy

$$U = m_0 \square$$

Which represents the contribution of the gravitational effect into the total energy.

CONCLUSION

The influence of gravity and motion on time, volume, and mass demonstrates their reliance on both potential and velocity. It is noteworthy that, when examining the influence of gravity just on mass as per equation (16), the mass exhibits a rise. This signifies that the field induces this increase. Conversely, temporal, volumetric, mass, and energy expressions demonstrating their reliance on gravitational potential via metric should now establish a valid correlation between Special Relativity and General Relativity. Since these expressions fundamentally rely on the metric component g_{00} within the weak field approximation, this scenario may permit an additional generalisation to encompass higher-order gravity. Consequently, at the threshold of an exceptionally strong gravitational field, when quantum effects are presumed to prevail, it is anticipated that these effects will result in a complex formulation of these statements. A complex expression delineating the gravitational influence on mass, time, and length derived from the generalised metric [8],[9]. Other than that, of General Relativity, it may unveil a novel concept regarding the properties of matter. The metric componentg_{00}, which characterises heavy gravity [10], would significantly augment the mass.

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