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RESEARCH ARTICLE

Planning of Full Factorial Experiments for the İnvestigation of Roughness in Hydroabrasive Waterjet Cutting of Hardox-500 Steel

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ABSTRACT

The article presents the mathematical planning of multifactor experiments to determine the mathematical dependence of the surface roughness on the cutting regime factors in the hydroabrasive waterjet cutting of HARDOX-500 chrome-nickel steel. A matrix of experiments was designed. At the same time, the regression coefficients determined from the mathematical dependence were calculated based on the values of the output parameters obtained from the experiments and presented in the mathematical equation of roughness.

INTRODUCTION

In the hydroabrasive waterjet cutting method, the planning of experiments is of great importance for studying the quantities of surface roughness, shape errors, microhardness, and other factors (output parameters) formed on the cut surface during the processing, depending on the regime parameters (input parameters). The requirements for the input parameters and the multifactorial solution of the experiment depending on them have been studied by many authors in planning the experiments [1-5].

In the article, the calculation and evaluation of the mathematical dependencies between input and output parameters in the technological process of manufacturing complex-profile parts from HARDOX-500 chrome-nickel steel using the waterjet cutting method were carried out. In the waterjet cutting method, the input parameters include water jet pressure (P, MPa), hardness of abrasive grains (T), abrasive consumption (Q, q_z), longitudinal feed rate (S_{long}), and the thickness of the material being processed (b, mm) while the output parameters studied are surface roughness (R_a, R_z, μm), dimensional errors (Δ, μm), microhardness (H_μ,kgf/mm^2), and other factors. The limits of the input parameters during the experiments were determined based on literature and production experience [2-3].

Purpose of the work. Multifactorial planning of experiments was carried out to develop a mathematical equation for the dependence of the average roughness value R_a on the cutting regime parameters in the waterjet cutting of HARDOX-500 sheet metal.

Scientific novelty of the work. A multifactorial mathematical planning method was used to study the effect of the regime parameters of the technological process on the average roughness value in the hydroabrasive waterjet cutting of chrome-nickel steels, such as HARDOX-500. In the full factorial planning of the experiments, the values of the output parameter, i.e., R_a, were studied within the intervals of 250÷350 MPa for the pressure of the water jet mixed with abrasive, 85÷215 g/l for the consumption of abrasive grains, and 27.4÷77.4 mm/min for the longitudinal feed rate S_{long}. As a result of the selected parameters, a mathematical model determining the roughness was developed, and the corresponding regression coefficients were found.

PLANNİNG OF MULTİFACTORİAL EXPERİMENTS İN HYDROABRASİVE WATERJET CUTTİNG

The planning of experiments has been carried out using matrices of the full factorial 2^3 type [2-3]. The intervals of variation for the input parameters in waterjet machining are provided in Table 1. Table 1 shows the coded levels of the factors.

Table 1. Coding of factors

Names and symbols of factors	Levels of variation			Variation range	Step of change
	-1	0	+1		
Water jet pressure P, MPa	250,0	300,0	350,0	100,0	50,0
Abrasive consumption of tools, Q, g/l	85	150	215	130	65
Longitudinal feed rate, S_{long}, mm/min	26,7	53,4	77,4	51,9	25,35

Experiments are conducted based on a planned design of 2^3 type. In this case, the number of factors (variables) is k=3, the number of levels is p=2, and the total number of experiments is $N=2^3$, which equals 8. We assume the experiments are repeated 3 times.The matrix for planning the experiment is given in Table 2.

Table 2. Matrix for planning the experiment

Experiment number	Natural values of the factors			Dimensionless values of the factors in the coordinate system				Output parameters
	Z_1	Z_2	Z_3	X_0	X_1	X_2	X_3	Y
1	250	85	26,7	+1	-1	-1	-1	4,524
2	350	85	26,7	+1	1	-1	-1	4,965
3	250	215	26,7	+1	-1	+1	-1	5,153
4	350	215	26,7	+1	1	+1	-1	5,417
5	250	85	77,4	+1	-1	-1	+1	3,994
6	350	85	77,4	+1	1	-1	+1	2,718
7	250	215	77,4	+1	-1	+1	+1	3,931
8	350	215	77,4	+1	1	+1	+1	3,072

After conducting the experiments, the results are subjected to statistical analysis. We calculate the variance of factors using the following formula, which determines the average quadratic deviation of factors.

$$S_i^2 = \frac{\sum_{i=1}^{m}(y_i - \bar{y}_i)^2}{m-1}, \qquad (1)$$

Here, $\bar{y}_i$ is the average arithmetic value of the factor obtained from three repeated experiments, expressed as follows.

$$\bar{y_i} = \frac{\sum_{i=1}^{3} y_i}{2}, \qquad (2)$$

To check the homogeneity of variance, we use the Cochran's criterion, i.e.

$$G = \frac{S_{max}^2}{\sum_{i=1}^{N} S^2}, \qquad (3)$$

To calculate the Cochran's criterion, we determine the average and maximum values of R_a based on 8 experiments with 3 repetitions.

$$S_{max} = R_{a.max} = 5{,}920\ \mu m$$

$$S_{average} = R_{a.average} = \left(\frac{\sum_1^2 R_a}{24}\right) = 4{,}524\ \mu m$$

$$G = \frac{5{,}920^2}{\sum_{i=1}^{N}(4{,}52)} = \frac{35{,}046}{163{,}44} = 0{,}214.$$

We compare the obtained result based on the table [3, p.77; 2, pp.59-60].

$G_{1-P}\ (f_1, f_2)$; p=0.95; degrees of freedom $f_1 = m$-1=3-1=2; $f_2 = N_2$=8.

If we consider that according to the table G_{table}=0,214, then $G<G_{table}$ condition is satisfied, therefore, the obtained results are adequate.

Thus, based on the matrix of the full factorial experiment accepted in hydroabrasive cutting, the mathematical expression of the experiments conducted can be stated as follows:

$$y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + \ldots + b_{12}x_1x_2 + b_{23}x_2x_3 + b_{123}x_1x_2x_3 \qquad (4)$$

here y=R_a; x are values of the input parameter, b is coefficient of polynomial regression.

We can determine the regression coefficients in full factorial experiments as follows:

$$b_i = \frac{\sum_{i=1}^{N} x_{yi} \cdot \bar{y_i}}{N}, \qquad (5)$$

To calculate the regression coefficients, we construct the planned matrix of the experiment. The natural and dimensionless values of the factors are shown in Table 2.

Calculation of the factor increments. [2, pp. 59-60]

$$Z_1^0 = \frac{250+350}{2} = 100; \quad Z_2^0 = \frac{85+215}{2} = 110$$

$$Z_3^0 = \frac{26{,}7+77{,}4}{2} = 51{,}9$$

$$\Delta Z_1 = \frac{350-250}{2} = 50$$

$$\Delta Z_2 = \frac{215 - 85}{2} = 65$$

$$\Delta Z_3 = \frac{77{,}4 - 26{,}7}{2} = \frac{50{,}7}{2} = 25{,}35$$

Let's calculate the regression coefficients based on formula (5).

$$b_0 = \frac{1}{8}\sum_{i=1}^{8} y_i = \frac{1}{8}(4{,}524 + 4{,}965 + 5{,}153 + 5{,}417 + 3{,}994 + 2{,}718 + 3{,}931 + 3{,}072)$$
$$= \frac{1}{8}\sum_{i=1}^{8} 33{,}774 = 4{,}222 \quad (6)$$

$$b_1 = \frac{1}{8}\sum_{i=1}^{8} y_i = \frac{1}{8}(-1 \cdot 4{,}524 + 1 \cdot 4{,}965 - 1 \cdot 5153 + 1 \cdot 5{,}417 - 1 \cdot 3{,}994 + 1 \cdot 2{,}718 - 1 \cdot 3{,}981$$
$$+ 1 \cdot 3{,}072) = \frac{1}{8}\sum_{i=1}^{8} -1{,}43 = 0{,}18 \quad (7)$$

$$b_2 = \frac{1}{8}\sum_{i=1}^{8} y_i = \frac{1}{8}(-1 \cdot 4{,}524 - 1 \cdot 4{,}965 + 1 \cdot 5153 + 1 \cdot 5{,}417 - 1 \cdot 3{,}994 - 1 \cdot 2{,}718 + 1 \cdot 3{,}981$$
$$+ 1 \cdot 3{,}072) = \frac{1}{8}\sum_{i=1}^{8} 1{,}37 = 0{,}172 \quad (8)$$

$$b_3 = \frac{1}{8}\sum_{i=1}^{8} y_i = \frac{1}{8}(-1 \cdot 4{,}524 - 1 \cdot 4{,}965 - 1 \cdot 5153 - 1 \cdot 5{,}417 + 1 \cdot 3{,}994 + 1 \cdot 2{,}718 +\cdot 3{,}981 + 1$$
$$\cdot 3{,}072) = \frac{1}{8}\sum_{i=1}^{8} -6{,}87 = -0{,}86 \quad (9)$$

We calculate the other regression coefficients (b_{12}, b_{13}, b_{23}, b_{123}) using the following formulas [2].

$$b_{12} = \frac{\sum_{i=0}^{N}(x_1 x_2) y_1}{N}; \qquad b_{13} = \frac{\sum_{i=0}^{N}(x_1 x_3) y_1}{N}; \quad (10)$$
$$b_{23} = \frac{\sum_{i=0}^{N}(x_2 x_3) y_1}{N};\ b_{123} = \frac{\sum_{i=0}^{N}(x_1 x_2 x_3) y_1}{N}$$

Here, x_1 , x_2 , x_3 are the dimensionless values of the factors; y_i –output parameters;

b_j – regression coefficient.

Based on the table [3,p.78; table 4.3], we calculate the coefficients b_{12}, b_{13}, b_{23}, b_{123} as follows:

$b_{12} = \frac{1}{8}\sum_{i=0}^{n} x_1 x_2 y_i = \frac{1}{8}(4{,}524 - 1 \cdot 4{,}965 - 1 \cdot 5{,}153 + 1 \cdot 5{,}417 + 1 \cdot 3{,}994 + 1 \cdot 2{,}718 - 1 \cdot 3{,}981 + 3{,}072) = \frac{1}{8}(-0{,}03) = -0{,}004$;

$b_{13} = \frac{1}{8}\sum_{i=0}^{n} x_1 x_3 y_2 = \frac{1}{8}(4{,}524 - 1 \cdot 4{,}965 + 5{,}153 - 1 \cdot 5{,}417 - 1 \cdot 3{,}994 + 2{,}718 - 1 \cdot 3{,}981 + 3{,}072) = \frac{1}{8}(-2{,}84) = -0{,}355$;

$b_{23} = \frac{1}{8}\sum_{i=0}^{n} x_2 x_3 y_i = \frac{1}{8}(4{,}524 + 4{,}965 - 1 \cdot 5{,}153 - 1 \cdot 5{,}417 - 1 \cdot 3{,}994 - 1 \cdot 2{,}718 + 3{,}981 + 3{,}072) = \frac{1}{8}(-0.79) = -0.1$;

$b_{123} = \frac{1}{8}\sum_{i=0}^{n} x_1 x_2 x_3 y_i = \frac{1}{8}(-1 \cdot 4{,}524 + 4{,}965 + 5{,}153 - 1 \cdot 5{,}417 + 1 \cdot 3{,}994 - 1 \cdot 2{,}718 - 1 \cdot 3{,}931 + 3{,}072) = \frac{1}{8}(0.594) = 0.075$.

For a three-factor experiment, the regression equation obtained will be as follows:

$$y(x_1 x_2 x_3) = 4{,}222 - 0{,}18x_1 + 0{,}172x_2 - 0{,}86x_3 - 0{,}004x_1x_2 - 0{,}355x_1x_3 - 0{,}1x_1x_2x_3 \quad (11)$$

The natural values of the factors contributing to surface roughness dependence will be as follows:

$$z_1 = x_1^{max} = 350;\ x_1^{min} = 250;\ x_1^0 = \frac{350+250}{2} = 300;\ \Delta x_1^0 = \frac{350-250}{2} = 50;$$

$$z_2 = x_2^{max} = 215;\ \ x_2^{min} = 85;\ x_2^0 = \frac{215+85}{2} = 100;\ \ \Delta x_2^0 \frac{215-85}{2} = 65;$$

$$z_3 = x_3^{max} = 77{,}4;\ \ x_3^{min} = 26{,}7;\ x_3^0 = \frac{77{,}4+26{,}7}{2} = 51{,}9;\ \Delta x_3^0 = \frac{77{,}4+26{,}7}{2} = 25{,}35.$$

Let's consider the obtained regression equation as follows for the natural values of the factors:

$$R_a(P, Q, S_{long}) = b_0 - 0{,}18 \cdot \frac{p^{max} - p^0}{\Delta p_b^0} + 0{,}172\frac{Q^{max} - Q^0}{\Delta Q} - 0{,}86\frac{S^{max} - S^0}{\Delta S^0} - 0{,}004 \cdot \frac{p^{max} - p^0}{\Delta p_b^0} \cdot \frac{Q^{max} - Q^0}{\Delta Q} - 0{,}355 \cdot \frac{p^{max} - p^0}{\Delta p_b^0} \cdot \frac{S^{max} - S^0}{\Delta S^0} - 0{,}1\frac{p^{max} - p^0}{\Delta p_b^0} \cdot \frac{Q^{max} - Q^0}{\Delta Q} \cdot \frac{S^{max} - S^0}{\Delta S^0} \quad (12)$$

Below is the calculation of the surface roughness corresponding to the maximum and minimum values of input factors:

$$R_a(P, Q, S_{long}) = 4{,}222 - 0{,}18\frac{p^{max} - 300}{50} + 0{,}172\frac{Q^{max} - 150}{65} - 0{,}86\frac{S^{max} - 53.4}{25.35} - 0{,}004 \cdot \frac{p^{max} - 300}{50} \cdot \frac{Q^{max} - 150}{65} - 0{,}355 \cdot \frac{p^{max} - 300}{50} \cdot \frac{S^{max} - 53.4}{25.35} - 0{,}1\frac{p^{max} - 300}{50} \cdot \frac{Q^{max} - 150}{65} \cdot \frac{S^{max} - 53.4}{25.35}$$

$R_a{}^{max}(P, Q, S_{long}) = 4{,}222 - 0{,}18 \cdot \frac{350-300}{50} + 0{,}172\frac{215-150}{65} - 0{,}86\frac{77.4-53.4}{25.35} - 0{,}004 \cdot \frac{350-300}{50} \cdot \frac{215-150}{65} - 0{,}355 \cdot \frac{350-300}{50} \cdot \frac{77.4-53.4}{25.35} - 0{,}1\frac{350-300}{50} \cdot \frac{215-150}{65} \cdot \frac{77.4-53.4}{25.35} = 4{,}222 - 0{,}18 \cdot 1 + 0{,}172 \cdot 1 - 0{,}86 \cdot 0.95 - 0{,}004 \cdot 1 \cdot 1 - 0{,}355 \cdot 1 \cdot 0{,}95 - 0{,}1 \cdot 1 \cdot 1 \cdot 0.95 = 4{,}222 - 0{,}18 + 0.172 - 0{,}817 - 0.004 - 0{,}337 - 0{,}095 = 2{,}961\mu m$

Let's calculate the mean variance to evaluate the dispersion of the mathematical calculation:

$$S^2_{dispersion} = \frac{\sum_{i=0}^{n} S_i^2}{N} = \frac{3{,}638^2}{8} = 1{,}65 \quad (13)$$

$$S_{dispersion} = \sqrt{1{,}65} = 1{,}286$$

Let's check the adequacy of the regression equation based on the Fisher criterion [3, p.79].

$$F = \frac{S^2_{\text{adequate}}}{S^2_{\text{dispersion}}} \quad , \qquad (14)$$

here;

$$S^2_{\text{adequate}} = \frac{m \sum_{i-1(}^{N} \bar{y}_\iota - y_i)}{\text{N} - \text{L}} \qquad (15)$$

where $S^2_{adequate}$ stands for the adequate variance, L is the number of factors.

$$S^2_{\text{adequate}} = \frac{3 \cdot 0{,}84}{8-7} = 2{,}52$$

$$F = \frac{2{,}52}{1{,}65} = 1{,}52$$

When comparing the obtained results according to the respective table [3, p.86], with a confidence level of p=0.95; when the degree of freedom for dispersion adequacy is f_1=N-L=8-7 =1; and when the degrees of freedom is f_2 = N(m-1) = 8(3-1)=16, then F_{table} = 3,63 [3, p.86, table 4.10]. The computed result for the test statistic is $F_{computed}$ =1,52.

Therefore, $F_{computed} < F_{table}$, indicating that the model of the obtained regression is adequate.

CONCLUSİON: As a result of the multi-factor planning of experiments on the average roughness R_a of the cut surface in hydroabrasive cutting of HARDOX-500 steel, the mathematical equation and regression coefficients depending on the pressure of the waterjet, the consumption of abrasive grains, the change in the longitudinal feed rate have been determined.

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