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RESEARCH ARTICLE

Theoretical Demonstration and Application of the Expert Weighted Method

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ABSTRACT

The expert weighted method was proposed over a decade ago, but there was no theory to justify this method back then. This study aims to verify the expert weighted method with academic theories and to consolidate its theoretical basis. To mitigate the setbacks of the 5-point Likert scale, which only allows addition but not multiplication or division, this study adopts a 100-point scoring scale, and experts were invited to assess the importance of each variable (baseline=1) to generate a matrix. The importance matrix of each variable was obtained after multiplying the matrices. In the obtained matrix, the importance of each variable is no longer restricted by the determination of "higher/lower", and the proportional relationship between the importance of the variables can be shown. To better understand the overriding relationships between the variables, this study adopts the "mathematical induction" method to prove it. If the scores of each variable are identical, the greater the score variation of one variable, the less important the variable is. This is the theoretical proof of the innovative expert weighted method. The study findings discovered that the expert weighted method using 100-point scoring system can enhance the Likert scale. Furthermore, evaluating the internal variance between variables helps to clearly identify the importance between variables, allowing variables to be reordered based on their importance. It indicated that the expert weighted method has the effect of enhancing the discrimination of efficiency evaluation. Although the Likert scale is widely employed in quantitative scoring mechanisms, some scholars believe this scale is only suitable for ranking and do not provide accurate numerical values for evaluation. Therefore, the statistical limitations of the Likert scale need to be further examined when measuring the significance of multiple variables to improve the reliability of the scale.

INTRODUCTION

Bao (2013) proposed an innovative "expert weighted metho", in which experts are invited to rate the importance of each variable with a Likert scale (1-5 points) based on their professional knowledge. Afterward, the importance scores are summed to produce an importance analysis for each variable. Gliem and Gliem (2003) argued that the Likert scale can only be employed to differentiate between two ratings and that the ratings are not precise enough for multiplicative comparisons. Jamieson (2004) also stated that the Likert scale cannot distinguish the importance of values, so the obtained mean and standard deviation are not suitable as a basis for ranking values. Additionally, Gardner and Martin (2007) pointed out that Likert scales are only suitable for ranking and cannot be utilized to compare numerical values. Similarly, Norman (2010) suggested that the Likert scale can be used for ranking but not for statistical analysis. Furthermore, some studies have indicated that although Likert scale is commonly used for quantitative scoring, it is only suitable for ranking rather than precise numerical assessment (Suh & Shin, 2019; Jones & Marshall, 2022). Therefore, the statistical limitations of the Likert scale in measuring the significance of the multivariate items need to be further examined in order to improve the reliability of this method. Li (2013) indicated that the Likert scale has equal consecutive intervals, which may lead to imprecise measurements. Sullivan and Artino (2013) believed that the Likert scale can only be used for ranking, but cannot be used to determine numerical relationships. Hartley (2014) argued that Likert scale is useful for ranking purposes, but the mean and standard deviation of ratings are not suitable for other analyses.

This study summarizes the above literature on the application of the Likert scale and provides the following explanation:

For example, experts were invited to rate the importance of each variable. If the full score is 100 and the score is divided into five equal intervals, it can be divided into "1-20", "21-40", "41-60", "61-80" and "81-100". If the 5-point Likert scale is adopted, the importance scores are rated as 1, 2, 3, 4 and 5. If these two scoring methods are used, the following situations can be drawn:

(a) When using a Likert scale, the first variable has a total score of 3 and the second variable has a total score of 3. When using a 100-point scoring system, the total score for the first variable is 60 and the total score for the second variable is 41. It can be observed that there is a significant difference in the scores obtained when using either the 5-point Likert scale or the 100-point scale. This is because the Likert scale only roughly represents the scores of each variable. If in-depth analyses need to be performed, it is inevitable that some irrational situations will occur.

(b) If the Likert scale is used, the scores of the first and the second variables add up to the same total, so the two scores cannot be compared. However, if a 100-point scoring system is employed, we can not only compare the sum of two variable scores, but also calculate the number of variations in each variable score.

In order to overcome the limitation that the Likert scale can only be used for summing and not suitable for division, this study adopted a two-stage scoring method. Firstly, experts were asked to rate the relative importance of each variable based on their expertise (with "1" as the base value) and form matrix β. The expert can then score the variables using a "100-point" scoring scale and create matrix X. After multiplying these two matrices, the variable importance matrix T is obtained. Finally, by dividing the "importance" of each variable i in the matrix T by the "importance" of the other variables *j*, the degree of "overriding" of variable *i* over variable *j* can be calculated.

Here is how to calculate relative importance when assessing *s* variables. First, divide each variable *j* in matrix T by *i* to generate *s* relative importance matrices $\hat{T}$ for $i = 1,\ldots,s$, $\mathrm{j} = 1,\ldots,n$. Next, combine the n^th^ column elements from these *s* matrices $\hat{T}$, arrange them based on matrix T^*, and transpose the result into matrix T^{r}. Finally, divide T^* by T^{r} to produce T^*T^r. *This* T^*T^r matrix reveals the overriding relationships between the variables by showing how much one variable outweighs another.

The results demonstrate that the expert weighted method not only optimizes the scoring mechanism of the Likert scale, but also shows the multiplicative relationship between variables. This method also allows for a more in-depth assessment of variables by analyzing internal variance, thereby increasing the degree of discrimination between variables. This paper is divided into five sections: Section 2 illustrates the principles of the expert weighted method; Section 3 elaborates on the innovations of the study, including a modified version of the Likert scale and the use of mathematical generalization to confirm the relationship between the degree of importance of the variables with the same total score and their variances; Section 4 provides examples; and Section 5 presents the conclusions.

Experts weighted method

When Bao (2013) proposed the expert weighted method, he emphasized that scoring must be carried out by representative, professional and authoritative experts. It is imperative that they have a comprehensive understanding of the scoring criteria and provide reasonable scores within the given range. The rationale for this approach is to allow experts to assess the relative importance of each variable based on their own professional judgment. Next, calculate the importance of variable *i* divided by the importance of variable *j* to determine the overriding relationships of variable *i* over variable *j*. The approach is also employed to ascertain the importance of each variable. In the event that there are *s* variables, the average importance of variable *i* can be calculated based on the ratio of these *s* variables. This method has been extensively utilized in the domain of multivariate assessment, exhibiting remarkable accuracy and reliability across a spectrum of applications. Recent studies have demonstrated the efficacy of the expert weighted method in multivariate assessment, particularly in the application of scoring matrices (Liu & Wang, 2022; Liu et al., 2020).

$$I = \begin{bmatrix} I_{11} & I_{12} & \bullet & \bullet & \bullet & I_{1s} \\ I_{21} & I_{22} & & & & I_{2s} \\ \bullet & & & & & \bullet \\ \bullet & & & & & \bullet \\ \bullet & & & & & \bullet \\ I_{n1} & I_{n2} & \bullet & \bullet & \bullet & I_{ns} \end{bmatrix}$$

The term "overriding" is defined as follows: The ratio obtained by dividing the importance of variable *i* by the importance of variable *j* is referred to as the degree of overriding of variable *j* by variable *i*.

If the ratio of the importance of variable *i* to that of variable *j* is > 1, it can be concluded that variable *i* overrides variable *j*, and thus the importance of variable *i* is higher than that of variable *j*. Conversely, if the ratio is < 1, it can be concluded that variable *i* is less important than variable *j*. The calculation steps are shown below:

1. To determine the relative importance of the *s* variables, experts were invited to rate them on a 5-point Likert scale. The resulting data are then employed to build the importance matrix I.

2. Divide the variables in row *j* of matrix I by variable i in other rows to obtain a relative importance matrix. Subsequently, the *n* column elements of these matrices are aggregated and sorted to form a matrix *T*, as follows:

$$T = \begin{bmatrix} A_1 & A_2 & \bullet & \bullet & \bullet & A_s \\ B_1 & B_2 & \bullet & \bullet & \bullet & B_s \\ \bullet & & & & & \\ \bullet & & & & & \\ \bullet & & & & & \\ S_1 & S_2 & \bullet & \bullet & \bullet & S_s \end{bmatrix}$$

The first column elements of matrix T was the sum of the columns obtained by dividing the first column element of matrix *I* by the sum of other row elements; Data for other columns were derived similarly. The values given by t_{ij} in each column represent the importance ratio of the i^{th} variable to the j^{th} variable. A higher t_{ij} indicated a greater overriding degree of the i^{th} variable over the j^{th} variable; and t_{rj} signified the degree of "override" of the r^{th} variable by the j^{th} variable, a larger t_{rj} showed that the importance of the j^{th} variable was higher than that of the r^{th} variable.

3. Because the r^{th} variable has some degree of override (t_{rj}) over the j^{th} variable, there is also a degree of override (t_{jr}) by the j^{th} variable. Thus, in the vector $[\frac{t_{i1}}{t_{1i}}, \frac{t_{i2}}{t_{2i}}, \ldots, \frac{t_{is}}{t_{si}}, i = 1,\ldots,s, n = 1,\ldots,n]$, each element is expressed as the overriding degree of variable i over variable j divided by the degree of variable i being overridden by variable j. The sum of the elements in the i^{th} row represents relative importance of the i^{th} variable in comparison with the other variables. The larger this value is, the more important the i^{th} variable is in relation to the other variables.

RESEARCH METHOD

Innovative design of the study

The methodology proposed by Bao et al. (2013) employed the traditional Likert scale to construct a questionnaire and analyze the importance of each question. However, the traditional Likert scale yielded identical values for the attitude measured for each variable, which is an issue that requires further investigation. The aggregation of the raters' importance scores for a variable yielded its degree of importance. To facilitate the utilization of the rated scores for both addition and multiplication, a minor modification was made to the Likert scale method. To address the constraints of the Likert scale on multivariate measures, fuzzy set theory has been introduced as an improvement in recent years, which can effectively enhance scoring accuracy and information content (Vonglao, 2017; Chen & Yu, 2020). The application of mathematical generalization has been shown to be efficient and accurate in calculating multivariate weights for expert evaluation methods (Zhang & Yu, 2021).

In this research, mathematical induction was employed for theoretical validation with the objective of reinforcing the innovative theoretical foundation of the expert-weighted method. The Likert scale was modified to allow the addition and multiplication of single values to calculate scale scores. When evaluating the importance of variables, each respondent is permitted to assign an importance weight to each variable based on their own judgment, as outlined in Table 1. Furthermore, they are able to determine the importance of each variable by adopting a 100-point scale. This scoring method allows respondents to express their judgments of the importance of the variables in a more flexible and precise manner, thereby reflecting the relative importance of each variable with greater accuracy.

A revision of the traditional Likert scale assessment

The majority of research evaluating variable importance relies on the conventional Likert scale as their primary assessment tool. This approach involves summing expert ratings for each variable, with higher totals indicating greater importance. Although this type of assessment method is relatively simple, it comes with several significant limitations:

(1) Due to the fact that the conventional Likert scale is only capable of measuring addition, not multiplication, it is also referred to as an aggregate scale. Consequently, a comparison of the aggregate scores of variables A and B allows for an approximate distinction between high and low scores, but not a precise measurement of the relative importance of these two variables.

(2) When the aggregate scores of multiple variables are frequently the same, this assessment method cannot effectively compare the importance of the variables.

(3) Aggregating scores can only obtain the average value, but cannot obtain the internal variation of the variables. In the event that the mean difference is not substantial, the variance would serve to influence the importance of the variable.

In order to improve the accuracy of variable importance assessment and enable aggregation and multiplication of scale scores, this research introduces an upgraded assessment method. The evaluation was conducted by a panel of experts, selected for their representative, authoritative, earnest, and responsible credentials. These experts were tasked with assessing the importance of the variables in question, assigning proper scores within a specified range. The experts were requested to specify the weights of the variables' importance to establish matrix β. Subsequently, the experts evaluated the importance of the variables by assigning them reasonable scores on a 100-point scale. These scores were employed to build matrix X, and the variables in β were multiplied with those of X to obtain matrix T.

The following scenario was presented: A manager of a restaurant attempted to assess the relative importance consumers place on price, hygiene, transportation, atmosphere, and taste. To this end, the following steps were performed:

Step 1: Ten experts were invited to assess the importance of four factors: hygiene, transportation, atmosphere, and taste. In order to provide a basis for comparison, price is used as a baseline. Table 1 lists the final ratings.

Table 1. Importance rated by experts for hygiene, transportation, atmosphere, and taste (with price as the baseline)

Expert	Price	Hygiene	Transportation	Atmosphere	Taste
1	1	1.1	1.1	0.9	1.1

2	1	1.2	1.1	1	1
3	1	1.1	1.2	1.2	1.1
4	1	1	1.1	0.9	1.1
5	1	1.2	1.1	1.2	1.1
6	1	1.1	1.1	1.1	1.2
7	1	1.2	1.1	1.1	1.2
8	1	1.2	1.1	1.2	1
9	1	1.1	1.2	1.1	1.1
10	1	1.1	1.2	1	1

These ratings were then utilized to build matrix β:

$$\text{Let } \beta = \begin{bmatrix} 1 & 1.1 & 1.1 & 0.9 & 1.1 \\ 1 & 1.2 & 1.1 & 1 & 1 \\ 1 & 1.1 & 1.2 & 1.2 & 1.1 \\ 1 & 1 & 1.1 & 0.9 & 1.1 \\ 1 & 1.2 & 1.1 & 1.2 & 1.1 \\ 1 & 1.1 & 1.1 & 1.1 & 1.2 \\ 1 & 1.2 & 1.1 & 1.1 & 1.2 \\ 1 & 1.2 & 1.1 & 1.2 & 1 \\ 1 & 1.1 & 1.2 & 1.1 & 1.1 \\ 1 & 1.1 & 1.2 & 1 & 1 \end{bmatrix}$$

Step 2: Request the experts provide ratings on a 100-point scale, based on their professional expertise (Table 2).

Table 2. Price rated by experts using a 100-point scale

	Price
1	58
2	73
3	58
4	58
5	79
6	58
7	58
8	57
9	76
10	75

These ratings were utilized to build matrix X.

$$\text{Let } X = \begin{bmatrix} 58 \\ 73 \\ 58 \\ 58 \\ 79 \\ 58 \\ 58 \\ 57 \\ 76 \\ 75 \end{bmatrix}$$

Step 3: Using the price as the baseline, multiply the matrix β variables by the variables of matrix X. It should be noted that the multiplication represented by "0" is not a typical matrix multiplication; rather, it represents the corresponding element multiplication of the elements of X with the elements of each row of β. The resulting product, matrix T, represents the importance of each variable. The established matrix demonstrated the importance of each relevant variable, as shown in Table 3.

Table 3. Multiply the matrix β variables by the variables of matrix X to establish matrix T (with price as the baseline)

$$T = X\beta = \begin{bmatrix} 58 & 63.8 & 63.8 & 52.2 & 63.8 \\ 73 & 87.6 & 80.3 & 73 & 73 \\ 58 & 63.8 & 69.6 & 69.6 & 63.8 \\ 58 & 58 & 63.8 & 52.2 & 63.8 \\ 79 & 94.8 & 86.9 & 94.8 & 86.9 \\ 58 & 63.8 & 63.8 & 63.8 & 69.6 \\ 45 & 69.6 & 63.8 & 63.8 & 69.6 \\ 57 & 68.4 & 62.7 & 68.4 & 57 \\ 76 & 83.6 & 91.2 & 83.6 & 83.6 \\ 75 & 82.5 & 90 & 75 & 75 \end{bmatrix}$$

The results demonstrated the overriding relationships between variables and confirm the existence of multiple relationships. Furthermore, mathematical induction was employed to substantiate the hypothesis that the degree of importance decreases with an increase in variance between variables.

Mathematical induction

During the expert assessment process, a multitude of variables received the same overall score. To elucidate the implications associated with each variable, this study employed the variance observed for each variable. Theorem 1 showed that the importance ratings of variables with the same aggregated scores are related to their variances.

Prior to establishing Theorem 1, it is essential to describe the concept of an ideal variable. An ideal variable can be defined as a variable for which all experts have assigned an identical importance score. This implies that all experts have reached a consensus regarding the significance of the variable in question.

Theorem 1:

If the total importance scores of variables A and B are identical, but the variance of the importance score of variable A is higher than that of variable B, then the importance of variable A is lower than that of variable B.

Proof:

Suppose *n* experts were invited to provide importance scores for A and B, which we may denote as $a_1, a_2, a_3, \ldots, a_n$ and $b_1, b_2, b_3, \ldots, b_n$, respectively. Furthermore, assume that the variance of variable A is greater than the variance of variable B, and that the mean score $\bar{a}$ of variable A is identical with the mean score $\bar{b}$ of B.

When the scores of variables A and B are respectively rearranged in descending and ascending order (i.e., $a_1 \geq a_2 \geq a_3 \geq \ldots \geq a_n; b_1 \leq b_2 \leq b_3 \leq \ldots \leq b_n$) to reflect their importance, then based on Theorem 1,

$$(a_1^2 + a_2^2 + a_3^2 + \ldots + a_n^2 - n\bar{a}^2) > (b_1^2 + b_2^2 + b_3^2 + \ldots + b_n^2 - n\bar{b}^2).$$

Which means,

$$(a_1^2 - b_1^2) + (a_2^2 - b_2^{2)}) + (a_3^2 - b_3^2) + \ldots + (a_n^2 - b_n^2) > 0.$$

Next, suppose an ideal variable is assigned an importance score of "q" by all experts. When its mean $q = \bar{a} = \bar{b}$ is divided by the importance scores of variables A and B, the relative scores of the ideal variable compared to variables A and B are demonstrated as below:

$\frac{q}{a_1}, \frac{q}{a_2}, \frac{q}{a_3}, \ldots, \frac{q}{a_n}$ and $\frac{q}{b_1}, \frac{q}{b_2}, \frac{q}{b_3}, \ldots, \frac{q}{b_n}$, respectively.

Therefore, equation (1) can be proven using mathematical induction:

$$\frac{1}{a_1} + \frac{1}{a_2} + \bullet\bullet\bullet + \frac{1}{a_n} > \frac{1}{b_1} + \frac{1}{b_2} + \bullet\bullet\bullet + \frac{1}{b_n} \quad (1)$$

1. For n=1, since $a_1^2 = b_1^2$, $\frac{1}{a_1} > \frac{1}{b_1}$, confirming that equation (1) holds.

2. Assume equation (1) holds when *n* = *k.* Thus, when

$$a_1^2 + a_2^2 + a_3^2 + \ldots + a_k^2 > b_1^2 + b_2^2 + b_3^2 + \ldots. + b_k^2,$$

$$(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \ldots + \frac{1}{a_k}) > (\frac{1}{b_1} + \frac{1}{b_2} + \frac{1}{b_3} + \ldots + \frac{1}{b_k}).$$

3. For *n* = *k* + 1, assume the total importance scores of variables A and B are equal.

In this case,

$$(a_{k+1} - b_{k+1}) < (a_k - b_{k+1}) < (a_k - b_k) <, \ldots, < (a_1 - b_1) < 0.$$

Therefore,

$$(a_{k+1} - b_{k+1}) < 0,$$

and consequently,

$$\frac{1}{a_{k+1}} > \frac{1}{b_{k+1}}$$

and

$$(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\ldots+\frac{1}{a_k}+\frac{1}{a_{k+1}}) > (\frac{1}{b_1}+\frac{1}{b_2}+\frac{1}{b_3}+\ldots+\frac{1}{b_k}+\frac{1}{b_{k+1}}) .$$

Thus, equation (1) holds.

4. Based on mathematical induction,

$$\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\ldots+\frac{1}{a_k}+\frac{1}{a_{k+1}} > \frac{1}{b_1}+\frac{1}{b_2}+\frac{1}{b_3}+\ldots+\frac{1}{b_k}+\frac{1}{b_{k+1}}$$

holds for any natural number *n*.

The proof of Theorem 1 indicates that a comparison of the scores of an ideal variable with those of A and B would yield the following total score:

$$(\frac{q}{a_1}+\ldots+\frac{q}{a_k}+\frac{q}{a_{k+1}})-(\frac{q}{b_1}+\ldots+\frac{q}{b_k}+\frac{q}{b_{k+1}}) > 0 \qquad (2)$$

In (2), $(\frac{q}{a_1}+\ldots+\frac{q}{a_k}+\frac{q}{a_{k+1}})$ represents a comparison of the scores of the ideal variable with those of variable A. The result exceeds the corresponding scores of the ideal variable divided by those of variable B ($\frac{q}{b_1}+\ldots+\frac{q}{b_k}+\frac{q}{b_{k+1}}$). Based on this comparison with the ideal variable, the importance of variable A is determined to be lower than that of variable B. Thus, the theorem is confirmed.

The proof of Theorem 1 highlights the necessity to consider both the total importance scores of each variable and their internal variance. Specifically, when the total scores of the variables are equal, the variance of the scores must be analyzed. Variables with higher variance are deemed less important than those with lower variance. Section 4 presents an example to demonstrate the expert-weighted method.

CASES ANALYSIS

Case 1:

Table 4-1 Importance Analysis of Service Items in a Restaurant

(Scored with Likert scale)

Expert	Price	Hygiene	Transportation	Atmosphere	Taste
1	3	3	3	3	3
2	4	3	4	3	4
3	3	4	3	4	4
4	3	4	3	3	4
5	4	3	3	3	4
6	3	4	3	4	4
7	3	3	3	4	3
8	3	4	4	3	2
9	4	3	4	4	3

10	4	3	3	3	3
Total	34	34	33	34	34

Table 4-2 Importance Analysis of Service Items in a Restaurant

(Scored with 100-point scoring system)

Expert	Price A	Hygiene B	Transportation C	Atmosphere D	Taste E
1	42	55	59	48	58
2	62	55	78	58	70
3	56	62	59	70	63
4	56	75	58	42	72
5	78	55	57	50	71
6	57	75	57	78	62
7	55	47	55	65	56
8	54	73	77	53	39
9	70	58	78	72	58
10	72	55	56	58	57
Total	602	610	634	594	606

As shown in the results of the analysis in Table 4-1, the following relationship can be obtained when using the Likert scale for the importance rating: Price = Hygiene = Atmosphere = Taste (34) > Transportation (33). However, the results of Table 4-2 show that using the "100-point scale", the importance relationships are: Transportation > Hygiene > Taste > Price > Atmosphere. The analyses above reveal that the utilization of the Likert scale has resulted in an obvious decline in the perceived importance of transportation (33) relative to other variables. Conversely, the application of the "100-point scale" has led to an increase in the perceived importance of transportation (634) when compared to other variables. This observation highlights the potential disadvantages of the Likert scale in its current scoring methodology.

Case 2: This section presents the scenario of improving the Likert scale and the application of the expert weighted method. The steps are as follows:

Step 1: Multiply the scoring matrix X in Table 3-2 by the scoring matrix β in Table 3-1 (using "price" as the baseline), and then obtain the matrix T, which represents the scoring results of the importance of each relevant variable; as shown in Table 4-1:

Table 4-3 Relative importance compared with other variables

(Scored with 100-point scoring system, with price as the baseline)

Expert	Price	Hygiene	Transportation	Atmosphere	Taste
1	58	63.8	63.8	52.2	63.8
2	73	87.6	80.3	73	73
3	58	63.8	69.6	69.6	63.8
4	58	58	63.8	52.2	63.8
5	79	94.8	86.9	94.8	86.9
6	58	63.8	63.8	63.8	69.6
7	45	69.6	63.8	63.8	69.6
8	57	68.4	62.7	68.4	57
9	76	83.6	91.2	83.6	83.6
10	75	82.5	90	75	75
Total	650	735.9	735.9	696.4	706.1

Table 4-3 illustrates the total scores for Hygiene and Transportation are identical; however, their calculated variances are 155.46325 and 146.6077, respectively. The overriding relationships between these variables demonstrate that Hygiene, which has a higher number of variables, is of lesser importance than Transportation. This is an exemplification of Theorem 1.

Consequently, the relative importance of the variables allowed not only a comparison of their scale but also facilitated calculations involving multiplication and division. Furthermore, the proportional relationships between the variables could be analyzed and compared. This method represents an improvement over the traditional Likert scale. However, as situations with equal scores may still occur, the proposed expert-weighted method incorporates mutual override relationships to calculate the scores for each variable. This allows for a clearer representation of the relative importance of individual variables.

As demonstrated in Appendix Table 8, the overriding relationships among the evaluated variables indicate that consumers' perceived importance of restaurants' five factors is ranked in the following order: Transportation > Hygiene > Taste > Atmosphere > Price.

CONCLUSIONS AND RECOMMENDATIONS

Bao et al. (2015) proposed the use of the experts weighted method but did not provide sufficient theoretical proof to support its efficacy. In order to establish and reinforce the theoretical basis of the proposed method, this study employed mathematical induction to illustrate that both the total scores and the internal variance of the variables must be taken into account when evaluating their significance. In particular, when the total scores of the variables are identical, their variances should be calculated, with a higher variance indicating a lower importance. Consequently, the 100-point expert weighted method enables comparison of score magnitudes and proportional relationships between variables. The relative importance of the variables can be determined by assessing their internal variance. The results of the previous case analysis are as follows:

(1) The research results indicated that when the Likert scale was employed to score the importance, the transportation variable (33) was rated as less important than other variables. However, when the "100-point scale" was used to score the importance, the transportation variable score (634) was rated as more important than other variables. This suggests that the Likert scale scoring method may be an unreliable measure of importance.

(2) The results showed that the total scores for Hygiene and Transportation at the restaurant were the same. However, upon calculating the variances for hygiene (155.46325) and transportation (146.6077), it was revealed that the overriding relationships indicated that hygiene, with a higher variance, was of lesser importance than transportation. This evaluation of internal variance effectively indicated the relative importance of the variables, aligning with the application of Theorem 1. The importance levels of the variables could be utilized to rank them. In conclusion, the expert weighted method can improve the differentiation efficiency in evaluation.

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APPENDIX

Step 1. Divide different baseline variables by other variables to demonstrate relative importance between variables. In the first step, the baseline variable is "Price", and the relative importance is calculated as below.

Table 1. The overriding relationships between variables (with "Price" as baseline)

Expert	Price	Hygiene	Transport-ation	Atmosphere	Taste
1	1	0.909091	0.909091	1.111111	0.909091
2	1	0.833333	0.909091	1	1
3	1	0.909091	0.833333	0.833333	0.909091
4	1	1	0.909091	1.111111	0.909091
5	1	0.833333	0.909091	0.833333	0.909091
6	1	0.909091	0.909091	0.909091	0.833333
7	1	0.646552	0.705329	0.705329	0.646552
8	1	0.833333	0.909091	0.833333	1
9	1	0.909091	0.833333	0.909091	0.909091
10	1	0.909091	0.833333	1	1
Total	10	8.692006	8.659875	9.245733	9.02534

Step 2. In this step, the baseline variable is "Hygiene", and the overriding relationships are shown below.

Table 2. The overriding relationships between variables (with "Hygiene" as baseline)

Expert	Price	Hygiene	Transport-ation	Atmosphere	Taste
1	1.1	1	1	1.222222	1
2	1.2	1	1.090909	1.2	1.2
3	1.1	1	0.916667	0.916667	1
4	1	1	0.909091	1.111111	0.909091
5	1.2	1	1.090909	1	1.090909
6	1.1	1	1	1	0.916667
7	1.546667	1	1.090909	1.090909	1
8	1.2	1	1.090909	1	1.2
9	1.1	1	0.916667	1	1
10	1.1	1	0.916667	1.1	1.1
Total	11.64667	10	10.02273	10.64091	10.41667

Step 3. The baseline variable in this step is "Transportation", and the overriding relationships are displayed below.

Table 3. The overriding relationships between variables (with "Transportation" as baseline)

Expert	Price	Hygiene	Transport-ation	Atmosphere	Taste
1	1.1	1	1	1.222222	1

2	1.1	0.916667	1	1.1	1.1
3	1.2	1.090909	1	1	1.090909
4	1.1	1.1	1	1.222222	1
5	1.1	0.916667	1	0.916667	1
6	1.1	1	1	1	0.916667
7	1.417778	0.916667	1	1	0.916667
8	1.1	0.916667	1	0.916667	1.1
9	1.2	1.090909	1	1.090909	1.090909
10	1.2	1.090909	1	1.2	1.2
Total	11.61778	10.03939	10	10.66869	10.41515

Step 4. In this step, the baseline variable is "Atmosphere", and the overriding relationships are demonstrated below.

Table 4. The overriding relationships between variables (with "Atmosphere" as baseline)

Expert	Price	Hygiene	Transport-ation	Atmosphere	Taste
1	0.9	0.818182	0.818182	1	0.818182
2	1	0.833333	0.909091	1	1
3	1.2	1.090909	1	1	1.090909
4	0.9	0.9	0.818182	1	0.818182
5	1.2	1	1.090909	1	1.090909
6	1.1	1	1	1	0.916667
7	1.417778	0.916667	1	1	0.916667
8	1.2	1	1.090909	1	1.2
9	1.1	1	0.916667	1	1
10	1	0.909091	0.833333	1	1
Total	11.01778	9.468182	9.477273	10	9.851515

Step 5. The baseline variable in this step is "Taste", and the overriding relationships between variables are shown below.

Table 5. The overriding relationships between variables (with "Taste" as baseline)

Expert	Price	Hygiene	Transport-ation	Atmosphere	Taste
1	1.1	1	1	1.222222	1
2	1	0.833333	0.909091	1	1
3	1.1	1	0.916667	0.916667	1
4	1.1	1.1	1	1.222222	1
5	1.1	0.916667	1	0.916667	1
6	1.2	1.090909	1.090909	1.090909	1
7	1.546667	1	1.090909	1.090909	1
8	1	0.833333	0.909091	0.833333	1
9	1.1	1	0.916667	1	1
10	1	0.909091	0.833333	1	1
Total	11.24667	9.683333	9.666667	10.29293	10

Step 6. Divide individual baseline variables by related variables, and aggregate the obtained scores (see the Table below):

Table 6. The scores aggregated after dividing each baseline variable by related variables

	Price	**Hygiene**	**Transport-ation**	**Atmosphere**	**Taste**
Price	10	8.692006	8.659875	9.245733	9.02534
Hygiene	11.64667	10	10.02273	10.64091	10.41667
Transport-ation	11.61778	10.03939	10	10.66869	10.41515
Atmosphere	11.01778	9.468182	9.477273	10	9.851515
Taste	11.24667	9.683333	9.666667	10.29293	10

Step 7. Transpose the matrix obtained in Step 6

Table 7. The Transposed Matrix of Table 6

Item	**Price**	**Hygiene**	**Transport-ation**	**Atmosphere**	**Taste**
Price	10	11.64667	11.61778	11.01778	11.24667
Hygiene	8.692006	10	10.03939	9.468182	9.683333
Transport ation	8.659875	10.02273	10	9.477273	9.666667
Atmosphere	9.245733	10.64091	10.66869	10	10.29293
Taste	9.02534	10.41667	10.41515	9.851515	10

Step 8: Divide the matrix in Appendix 6 by the matrix in Appendix 7 to obtain the overriding relationships between the variables, and then normalize the values for evaluation, as shown in the table below:

Table Appendix 8 Normalization results of mutual overriding relationships between variables

Item	**Price**	**Hygiene**	**Transport ation**	**Atmosphere**	**Taste**		
Price	1	0.785733	0.784393	0.88313	0.845149	4.298405	0.170558
Hygiene	1.272696	1	0.99834	1.12386	1.075731	5.470627	0.217071
Transport ation	1.274872	1.001663	1	1.125713	1.077429	5.479677	0.21743
Atmosphere	1.132336	0.889791	0.888326	1	0.957115	4.867567	0.193142
Taste	1.183224	0.9296	0.928135	1.044807	1	5.085765	0.2018