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RESEARCH ARTICLE

Model of Collaborative Learning for the Improvement of Formative Evaluation of Students in Mathematics in an Educational Institution

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ABSTRACT

Basic education from the circles of academic and social interaction and participation in virtual environments to develop cooperative learning in mathematics among secondary education stu-dents is increasingly interconnected in immersive and inclusive classrooms. On the other hand, methodological research and practical justification in these two educational aspects are not al-ways aligned with the same objective. In the context of existing research on basic educational pedagogical theory, paradigms, access concepts, and the non-experimental findings of a descrip-tive exploratory study, we develop how these two subfields of education perceive teaching ex-perience from a holistic approach. Mathematics teachers were asked to outline and prioritize various types of experiences necessary to systematically formulate mathematical questions with a clear purpose within an educational and social environment. The results revealed significant discrepancies between groups regarding the causal valuation of the importance of understand-ing individual student characteristics and overall experience in teaching and learning. Further-more, marked differences were identified in the perception and evaluation of the relevance of addressing the needs of students with disabilities and mastery of mathematical content. These differences are analyzed in depth, and cognitive strategies are proposed to strengthen profes-sional collaboration among educators.

INTRODUCTION

The general concept that basic education teachers reflect on and investigate their teaching and learning methods is a practice that is just beginning to consolidate in today's world. However, examining in detail and deeply the academic dynamics and internal structures of these basic education institutions can be uncommon, as they have traditionally focused on modifying strategies to improve student learning without applying similar processes of review and adjustment to their own practices and pedagogical systems in a holistic manner. This approach often overlooks the fact that the realities presented at the educational level in a general context are very different from those of teaching and learning in a specific context (Hao et al., 2022b).

In the field of basic education, specifically in regular education, it has been emphasized that there is a need to understand and address the interaction between mathematics teaching and basic education, for which an effective solution has yet to be found. This educational approach not only responds to the increasing diversity in classrooms but also recognizes that collaboration among pedagogical disciplines can be a powerful tool for improving educational outcomes. Graduate studies in education, particularly at the doctoral level with a foundation in specialized master's programs, explore how this interaction can promote more inclusive and effective strategies from a social and socio-critical perspective, allowing teachers to meet the needs of all students, regardless of their abilities or limitations related to origin, gender, reality, and prior cognitive processes (De Jong et al., 2005a).

In countries like China, Singapore, and some others in this part of the world such as the United States and Colombia, teacher training programs have begun to integrate mathematics courses into the preparation of future regular basic education teachers, and similarly, specialized basic education courses for future mathematics teachers, regardless of their background (Maldonado et al., 2023; Saleekongchai et al., 2024). This innovative and immersive approach seeks to ensure that new teachers are prepared to collaborate in inclusive classrooms, accommodating both general education students and those with demanding needs in specific occupational fields. However, this collaboration often faces barriers, creating significant gaps, as the perspectives and objectives between these two areas tend to differ significantly depending on culture, academic regime, labor incentives, and primarily teacher training (Corter et al., 2011; Jam et al., 2016).

Research in postdoctoral programs also highlights international examples, such as the case of the Nordic educational systems, particularly in countries like Finland and Denmark. These countries have demonstrated a remarkable ability to integrate interdisciplinary approaches in teaching, achieving high standards in mathematics while developing inclusive methods for students with profound collaborative and experiential learning needs (Rivera-Pérez et al., 2021). In these educational systems and models, cooperation between disciplines is based on a strong commitment to the continuous training of teachers and the application of cutting-edge educational research, associated with ongoing evaluation and teacher training. This educational model offers valuable lessons for South America, especially for underdeveloped countries with low educational levels according to global standards and recognized standardized metrics, where educational systems still face significant challenges in terms of inclusion and academic performance related to budget issues and basic teacher training (Corujo et al., 2020).

For the common denominator of the academic aspect, cooperative learning is one where students are oriented towards obtaining benefits for themselves and for the members of the group in general. This process involves students working together to maximize their own learning and that of the group, requiring simultaneous and proportional participatory work from each member. Based on this, fostering the integration of these approaches could transform the way teachers are trained and how learning is organized in the classroom, aimed at the ultimate goal of improvement. Cultural and economic differences should not be seen as limitations to developing optimal learning but rather as opportunities to adapt and apply successful models to local needs. Universities, institutes, and schools—all within the realm of education—play a crucial role in this process, as they can promote research that adapts successful practices to the Latin American context, respecting the cultural and social particularities of the region (Nolinske & Millis, 1999).

There are high-impact scientific articles that indicate how teacher educators in mathematics and basic education value different types of professional experience to prepare future teachers in these occupational fields. The results revealed significant discrepancies in the importance they attribute to knowledge of the individual student, previously gathered, general teaching experience, and mastery of mathematical content as an end result, especially concerning the needs of students with different epistemological backgrounds. These differences highlight the need for greater interdisciplinary collaboration and mutual understanding between these areas, which is a central and common issue in today's global educational landscape (Rahman & Lewis, 2020).

Furthermore, collaborative learning circles and the exchange of experiences allow for learning from practices in terms of curriculum planning, collaborative research, and the use of educational technology. This type of integration would not only improve the quality of education but also promote a more inclusive and equitable learning environment in the mathematical aspect for basic education students. Cognitive work in education, as spaces for innovation and critical analysis, can lead these changes by promoting an educational approach that combines principles of academic excellence with values of inclusion and equity. This includes the teacher's ability to build a set of social relationships between themselves and their students with the aim of carrying out a defined action within a specific academic time-space framework, ultimately intended to provoke positive changes in students and enhance knowledge retention (Bores-García et al., 2020).

Finally, the collaboration between mathematics teaching and basic education represents an invaluable opportunity to transform education for the benefit of students both individually and collectively. By combining best practices with the interpersonal needs and contexts of our reality, educational systems can move towards a more inclusive, innovative, and effective model capable of responding to the demands of a constantly evolving society. This allows for the creation of an organized educational dimension and a teaching competence based on academic and social participation for the benefit of students (Sahlberg, 2010).

2. MATERIALS AND METHODS

This research, focused on the field of basic education, has an applied nature and seeks to develop practical solutions to optimize collaborative learning circles based on academic and social participation, with the aim of strengthening cooperative learning in the area of mathematics for secondary education students. According to Manzano et al. (2022), this approach addresses the need to tackle specific issues in classrooms, promoting both academic progress and the development of social and collaborative skills. They utilized a quantitative approach with a non-experimental design, which implies that the variables were not manipulated nor were controlled conditions established during the study. Additionally, a cross-sectional design with a correlational-causal descriptive scope was used, allowing for the analysis of relationships between key variables at a specific moment without intervening in their natural development. This approach is particularly suitable for exploring how academic and social participation influences cooperative learning in mathematics, providing a detailed view of these relationships (EL-Deghaidy & Nouby, 2008).

Following the proposed methodology, this type of research facilitates the evaluation of a specific moment in the educational environment and the analysis of the association between variables, such as the impact of interaction circles on the improvement of cooperative learning. This allows for the identification of patterns and significant relationships without altering the natural behavior of the participants, contributing to a clear and useful diagnosis for designing future interventions (Pérez Sánchez & Poveda, 2014).

In this way, this research not only seeks to understand the current dynamics in the classroom but also to propose effective strategies that can be implemented by teachers to enhance learning in mathematics through collaborative and inclusive approaches.

In the context of this research, the population is defined as the group of students who share specific characteristics, serving as the main object of study. This aspect is fundamental for establishing the scope and relevance of the research, allowing for the identification and analysis of patterns and behaviors related to cooperative learning in the area of mathematics. From this perspective, the population includes 250 primary school teachers from an educational institution, from which a sample of 40 teachers was selected using non-probabilistic convenience sampling (Hao et al., 2019).

The study on the operationalization of the variables was framed in three main dimensions: planning, implementation of strategies, and verification of effectiveness in students. These dimensions were evaluated through a questionnaire with 40 items using Likert-type scales. The dependent variable, cooperative learning, focuses on fostering academic and social interaction among students, promoting a significant improvement in their performance in mathematics. Additionally, following the proposed methodology, three additional dimensions were included: knowledge of mathematical content, cooperative interaction skills, and teaching competencies in collaborative strategies, which were also evaluated using a questionnaire with Likert-type scales (Bunch, 2009).

In the current context of this research, the dimensions were designed to comprehensively analyze the impact of collaborative learning circles based on academic and social participation (Koutrouba et al., 2012). The planning dimension encompassed the structuring of activities designed to foster teamwork in virtual environments; the implementation evaluated the effectiveness of these strategies in classroom dynamics, and the verification measured the results obtained in terms of learning and social skills. Additionally, the study aimed to analyze the teacher's role in integrating these collaborative strategies, assessing their knowledge of mathematical content and their ability to promote inclusive and participatory learning environments. This cognitive approach allowed for a detailed diagnosis of the current state of cooperative learning in mathematics, providing a solid foundation for designing more effective pedagogical interventions tailored to students' needs (Astuti & Lammers, 2020).

Finally, this methodology not only allowed for the evaluation of the implementation of collaborative learning circles but also provided a replicable model that can be adopted by other educational institutions interested in improving cooperative learning and student participation in the area of mathematics.

Figure 1. Weighted Semantic networks of Collaborative Learning Circles with intention extracted from Atlas TI.

This modeling provides a comprehensive view of the interactions between the different stages, processes, and subprocesses of the didactic orientation model, facilitating its understanding and practical application to improve formative assessment in the educational context.

Figure 1 presents a weighted semantic network that describes the relationships between the key elements of the Collaborative Learning Circles model. This representative scheme combines a qualitative and quantitative approach, graphically representing the main nodes (dimensions and subdimensions) and their interrelations, where each connection includes a specific intent and a numerical weight indicating the relevance or strength of that connection.

Literal Description of the Network

The network is organized into three main dimensions, from which specific subdimensions derive. These dimensions are:

Planning (β): Focuses on the initial organization of learning through activities, needs analysis, and goal definition.

Implementation of Strategies (γ): Includes practical techniques to promote collaboration and the use of technological tools.

Verification of Effectiveness (δ): Evaluates the impact of strategies through the analysis of academic performance and the development of social skills.

Below, each dimension is detailed along with its corresponding subdimensions, intents, and weighted values.

Table 1 Relationships and Weights in the Semantic Network

Main Node	Sub node	Intent of the Relationship	Weight
Collaborative Learning Circles (α)	Planning (β)	Organize learning goals	0.80
Collaborative Learning Circles (α)	Strategy Implementation (γ)	Execute teaching strategies	0.90
Collaborative Learning Circles (α)	Effectiveness Verification (δ)	Measure learning outcomes	0.70
Planning (β)	Activity Design (β1)	Develop activities	0.60
Planning (β)	Needs Analysis (β2)	Identify educational gaps	0.80
Planning (β)	Objective Definition (β3)	Define measurable goals	0.90
Strategy Implementation (γ)	Teamwork Techniques (γ1)	Foster collaboration	0.70
Strategy Implementation (γ)	Technology Use (γ2)	Integrate digital tools	0.80
Strategy Implementation (γ)	Social Interaction Promotion (γ3)	Promote peer interaction	0.90
Effectiveness Verification (δ)	Academic Performance Assessment (δ1)	Evaluate academic progress	0.80
Effectiveness Verification (δ)	Cooperative Feedback (δ2)	Provide constructive feedback	0.70
Effectiveness Verification (δ)	Social Skills Impact (δ3)	Measure the development of social skills	0.90
Social Skills Impact (δ3)	Improved Math Learning (δ3.1)	Improve mathematical knowledge	0.95
Social Skills Impact (δ3)	Increased Motivation (δ3.2)	Increase student engagement	0.85
Social Skills Impact (δ3)	Enhanced Collaborative Skills (δ3.3)	Develop teamwork skills	0.90

Cognitive Interpretation of the Model

Intentions and Weights: The intentions specify the purpose of each relationship, while the weights (0.60, 0.60, 0.60 - 0.95, 0.95, 0.95) quantify the importance of the connections, indicating their relevance in the development of collaborative learning.

Integrated Dimensions: The relationships between the dimensions allow for a holistic analysis of the model, highlighting how planning, implementation, and verification interact to optimize learning in mathematics.

Educational Impact: This model demonstrates how the interaction between social, academic, and technological skills can strengthen student motivation and performance.

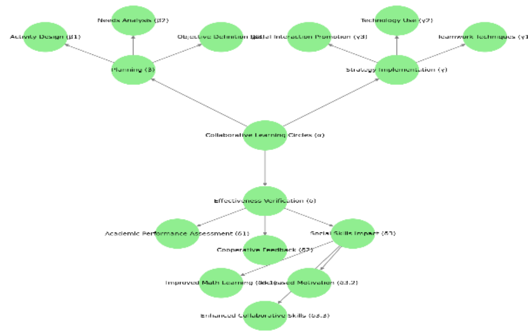


Figure 2. Mathematical Semantic networks of Collaborative Learning Circles extracted from Atlas TI.

Figure 2 represents a mathematical semantic network that illustrates the hierarchical relationships between the key concepts of Collaborative Learning Circles, organized with mathematical notations (α, β, δ) that identify the main dimensions and subdimensions. This structural design allows for a more technical and structured interpretation, facilitating its application in the educational field.

Semantic Structure of the Network

The network is divided into three main dimensions, each represented by a mathematical symbol, along with their respective subdimensions:

α: Collaborative Learning Circles (Central Dimension)

Subdimensions: Activity Design (β1), Needs Analysis (β2), Goal Definition (β3)

β: Implementation of Strategies

Subdimensions: Teamwork Techniques (γ1), Use of Technology (γ2), Promotion of Social Interaction (γ3)

δ: Verification of Effectiveness

Subdimensions: Evaluation of Academic Performance (δ1), Cooperative Feedback (δ2), Impact on Social Skills (δ3)

From these dimensions, specific educational intentions and numerical values are related, indicating the discretionary relevance of the interactions.

Table 2 Hierarchical and Mathematical Relationships

Main Node	Sub node	Educational Intent
α	β	Design and structure pedagogical objectives
	γ	Implement teaching strategies
	δ	Verify learning outcomes
β: Planning	β1: Activity Design	Create collaborative activities
	β2: Needs Analysis	Identify areas for improvement in the classroom
	β3: Goal Definition	Establish specific goals

γ: Implementation	γ1: Teamwork Techniques	Foster collaboration among students
	γ2: Use of Technology	Integrate digital tools
	γ3: Social Interaction	Promote peer interaction
δ: Verification	δ1: Academic Evaluation	Analyze academic progress
	δ2: Feedback	Provide constructive feedback
	δ3: Social Impact	Evaluate the development of social skills

Interpretation of the Mathematical Model

Dimensions and Relationships: The network shows how the components (α, β, γ, δ) interconnect to generate a comprehensive pedagogical model.

Conceptual Hierarchy: The subnodes allow for the breakdown of each dimension into specific tasks, providing a granular view of the educational process.

Technical Application: The use of mathematical notations organizes the structure in a systematic way, making it ideal for educational research that seeks a quantitative methodological approach.

Comparison with Figure 1

While Figure 1 includes numerical values and weighted relationships, Figure 2 focuses on a more structural and technical approach, where mathematical notations highlight the hierarchy and connection between concepts. Both figures are complementary, as they combine technical description with practical interpretation. This finding is important because it allows for a model without quantification, making it adaptable to any collaborative methodological process.

Everything developed prompts us to reflect on the role of basic education and the teaching of mathematics, which have developed differentiated approaches that, over time, have created significant distinctions in their practices and objectives. These differences are particularly evident in the way both disciplines address teaching, learning, and educational equity. In Li et al. (2024), we are guided to observe this research from the teaching experience and educational perspective in these fields, and based on the data from the previous figures, we can identify that planning, implementation of strategies, and verification of effectiveness are fundamental dimensions for understanding these divergences.

In the area of mathematics education, the traditional approach has focused on conceptual accuracy, mastery of content, and assessment of academic performance. On the other hand, basic education prioritizes social inclusion, curricular adaptation and design, and the development of social skills, emphasizing the importance of adjusting strategies to meet the individual needs of students in relation to their soft skills. This observation and difference in approaches can hinder the integration of collaborative practices in the classroom (Khalil & Aldridge, 2019).

However, by integrating and combining these perspectives within a cooperative learning model, as described in the previous figures, significant synergies can be generated. For example, in Öztürk and Korkmaz (2019), the planning dimension (β) allows for the establishment of clear objectives and the design of inclusive activities, while the implementation of strategies (γ) fosters social interaction and the use of technological tools. Finally, the verification of effectiveness (δ) ensures that both conceptual learning and personal development are evaluated comprehensively.

These differences, far from being a metacognitive obstacle, offer opportunities to build an educational model that integrates the best of both disciplines. In this context, Collaborative Learning Circles emerge as a practical solution to promote equity and improve learning in mathematics, adapting to the needs of all students and fostering collaboration in the classroom (Thibaut et al., 2018).

Data collection in this research was conducted using a structured and validated questionnaire, specifically designed to assess the proposed dimensions in the study on Circles of Interaction and Academic and Social Participation in Virtual Environments to Develop Cooperative Learning in the Area of Mathematics for Secondary Education Students. This questionnaire was created on the Google Forms platform and reviewed by a panel of experts, ensuring its validity and clarity for each of the items. The data collected were organized in Microsoft Excel and analyzed using SPSS software, allowing for the application of descriptive and inferential statistical techniques to ensure the validity of the proposed hypotheses.

The reliability of the instruments used was evaluated using Cronbach's Alpha coefficient. In terms of reliability, we chose not to work with the Aiken V, achieving high levels of internal consistency. In this case, the dimension of teaching orientation reached an $\alpha=0.934$, while formative assessment recorded an $\alpha=0.9$. These values indicate a high reliability of the variables, which supports the quality of the data collected for analysis.

The questionnaire consisted of a total of 40 items for each main variable, distributed in a way that covered all the defined dimensions and indicators. The key dimensions include planning, implementation of strategies, and verification of effectiveness, each with specific subdimensions. In the planning aspect, topics such as activity design, needs analysis, and goal definition were addressed (Faggiano et al., 2007). Additionally, the implementation of strategies focused on promoting teamwork, the use of technological tools, and social interaction. In another instance, the verification of effectiveness evaluated academic performance, cooperative feedback, and the impact on students' social skills. Among the most commonly used techniques were Pearson correlation, which allowed for the identification of significant relationships between key dimensions; independent T-tests, which analyzed differences between groups; and multiple regression models, which determined the weight of each dimension in the overall results of collaborative learning (Shah, 2023).

In summary, the approach adopted provided a comprehensive view of how collaborative learning circles can impact the development of cooperative learning in mathematics. The results and visualization of the analysis provide a solid foundation for designing effective pedagogical strategies that promote both academic participation and social development in secondary educational environments. This research process reinforces the relevance of the proposed model and its potential to transform classroom learning dynamics.

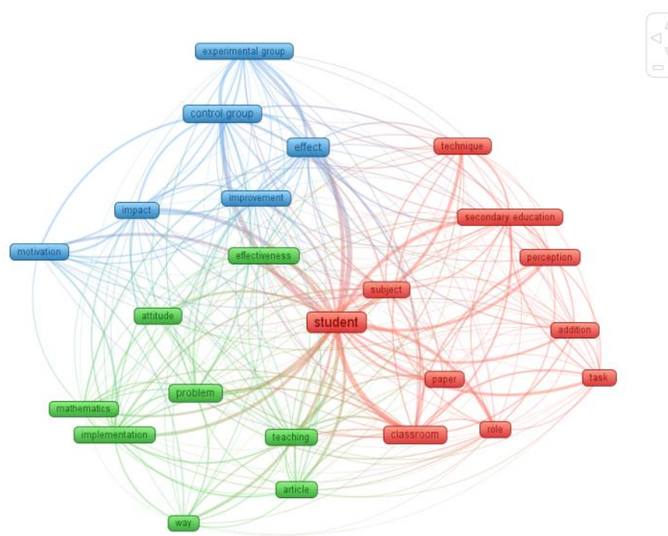


Figure 3. Bibliometric network extracted with VOS Viewer

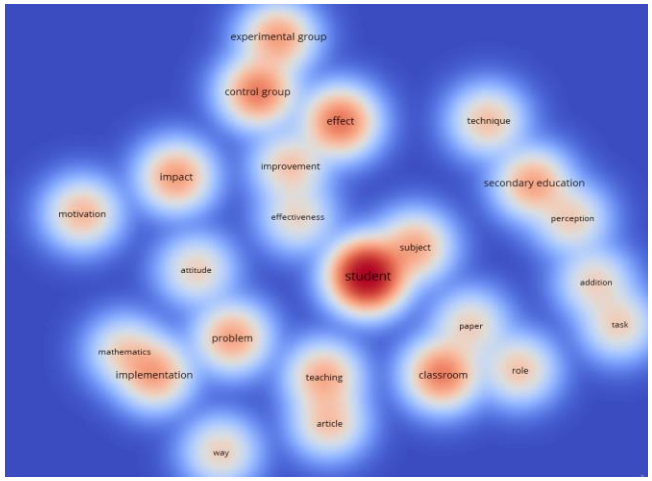


Figure 4. Density Visualization with VOS Viewer

The two images generated with VOSviewer represent visualizations related to key concepts in educational research, specifically regarding the impact of learning in mathematics within the context of secondary education. The first image uses a network map where the central node is the student, connected to keywords such as control group, experimental group, effectiveness, implementation, and secondary education. These connections reflect the centrality of the student in the analysis and how factors such as impact, motivation, and the implementation of strategies directly influence their learning. On the other hand, the second image presents a heat map of the same data, highlighting the relative importance of each concept through a color gradient: the most relevant concepts, such as student, effect, and classroom, are found in areas of higher intensity (red), while others, such as article and way, have secondary relevance with softer colors. Both representations reinforce the relationship between collaborative learning, effectiveness in strategy implementation, and the central role of the student as the focus of analysis in educational settings. This aspect emphasizes the importance of the student within the collaborative model, which has a direct connection to development, implementation, and mathematics as a learning system.

Table 3 Information on Gender and Teaching Experience of Participants

Category	**Mathematics (n = 14)**	**Basic Education (n = 40)**	**Both Areas (n = 25)**
Gender			
Male	11	14	6
Female	3	26	19
Teaching Experience*			
Early Childhood Education	2	12	4
Primary Education	4	15	10
Secondary Education	7	10	5
General Education	0	10	8
Learning Disabilities	0	6	5

Table 4 Mean and Standard Deviation of Ratings on Required Knowledge and Experience.

It is important:	Total (N = 40)	Mathematics (n = 14)	Basic Education (n = 40)	Both Areas (n = 25)
Have a deep and broad knowledge of mathematics	4.05 ± 0.90	4.10 ± 0.85	3.85 ± 1.00	4.00 ± 0.88
Have a general and holistic understanding of how students learn mathematics	4.45 ± 0.68	4.50 ± 0.60	4.40 ± 0.70	4.50 ± 0.65
Know students individually without segmentation	4.50 ± 0.65	4.20 ± 0.90	4.60 ± 0.50	4.55 ± 0.60
Understand the specific challenges of students with special abilities or particular difficulties	4.40 ± 0.75	3.95 ± 1.00	4.55 ± 0.65	4.50 ± 0.70
Have general teaching experience in basic education	3.70 ± 0.95	3.10 ± 0.90	4.00 ± 1.10	3.75 ± 0.90
Have experience teaching mathematics at different educational levels	3.70 ± 0.98	3.80 ± 0.70	3.50 ± 1.40	3.70 ± 0.95
Have experience in regular basic education	3.25 ± 1.10	2.80 ± 0.80	3.40 ± 1.50	3.30 ± 1.05

3. RESULTS

Descriptive Analysis of the Variables

Distribution of Collaborative Learning Dimensions

Figure 5 shows that, out of 40 teachers in an Educational Institution, presents the distribution of gender and teaching experience across three categories: **Math (n = 14)**, **Basic Education (n = 40)**, and **Both Areas (n = 25)**. It shows a clear predominance of females in **Basic Education** and **Both Areas**, whereas males are more evenly distributed across the categories, with a slight predominance in **Math**. Regarding teaching experience, **Primary Education** and **Secondary Education** are prominent in all groups, especially in **Basic Education**, which consistently has the highest counts. **General Education** and **Learning Disabilities** show moderate representation, with a smaller presence in **Math**. This visualization highlights the gender disparity and varying levels of expertise within different areas, reflecting the demographic and professional trends in the educational fields analyzed.

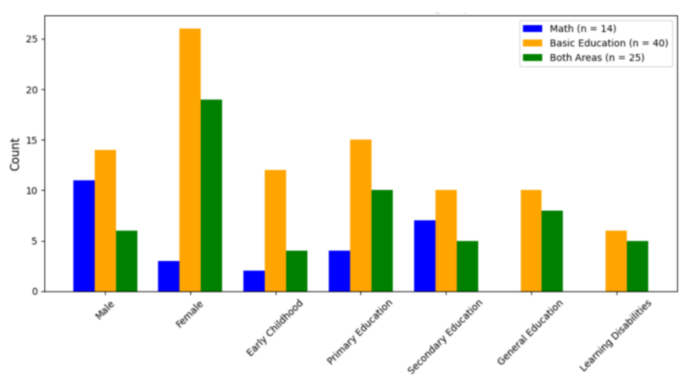


Figure 5. Percentage Distribution of the Collaborative Learning

Distribution of Collaborative Learning

In Figure 6, it is observed that, out of 40 teachers, the radar shows the comparison between the groups (Total, Mathematics, Basic Education, and Both Areas) across seven key dimensions of knowledge and educational experience. A high and uniform valuation is noted in the deep knowledge of mathematics and in the general understanding of how students learn mathematics, highlighting their cross-cutting importance. Dimensions such as individualized knowledge of students and the specific challenges faced by those with special needs are particularly valued by Basic Education and Both Areas, while Mathematics tends to prioritize experience in teaching at specific levels and technical knowledge. Experience in regular basic education receives lower scores, especially in Mathematics, indicating a less general context-focused approach. This analysis highlights a general alignment in the importance of key competencies but also reveals specific differences in the priorities of each group, underscoring the need for professional training programs tailored to the strengths and areas for improvement of each educational context.

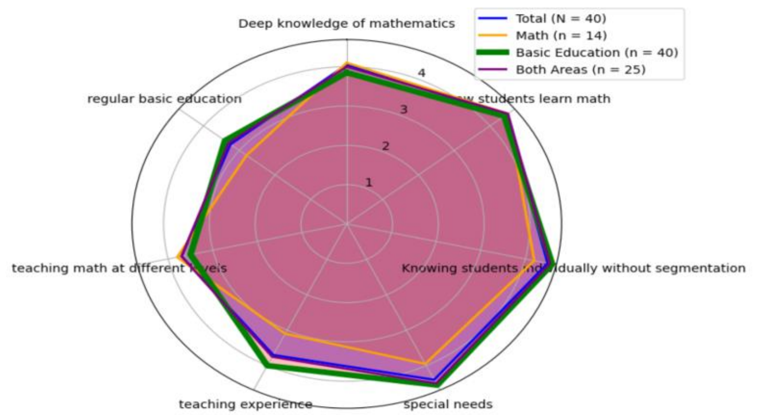


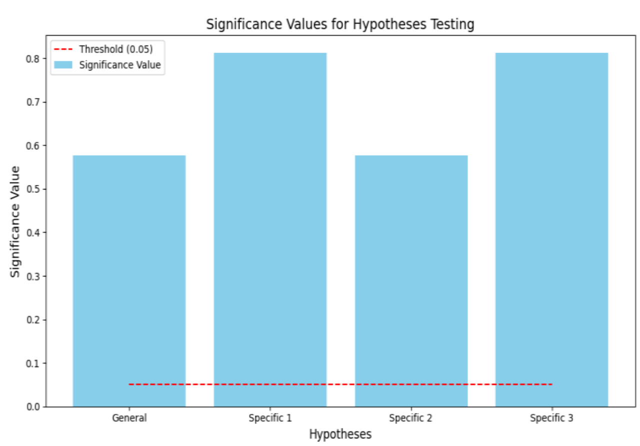
Figure 6. Percentage Distribution of the Collaborative Learning

Inferential analysis of the variables

General hypothesis testing

Table 5 shows that the significance is 0.576 > 0.05, so the alternative hypothesis is accepted. This means that there is no significant relationship between the variables Didactic Orientation and Formative Assessment; one does not significantly affect the other. Additionally, the relationship is 0.106, indicating a positive correlation between both.

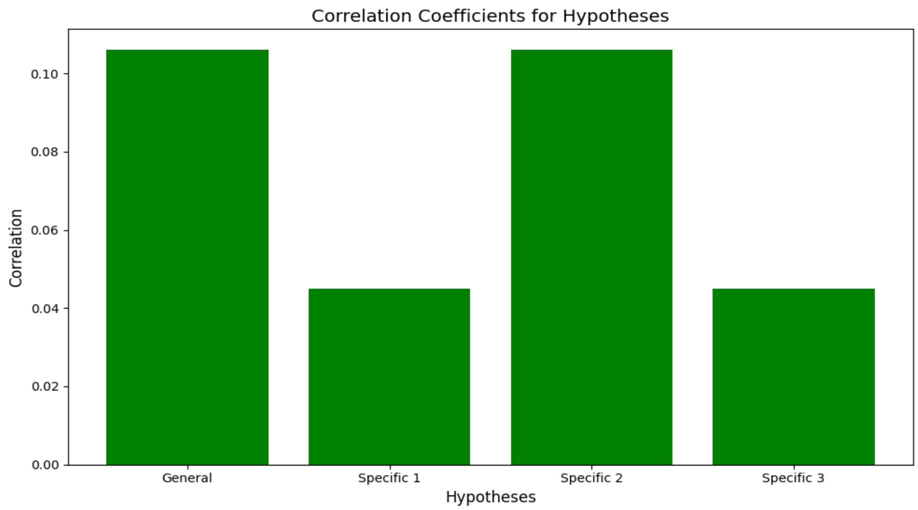
Table 5. General Hypothesis Testing



Specific Hypothesis Testing 1 and 2

Table 6 shows that the significance value is 0.812 > 0.05, accepting the alternative hypothesis. This indicates that there is no significant relationship between the dimension Implementation of Strategies and the variable Formative Assessment. The relationship is 0.045, reflecting a very low positive correlation.

Table 6. Specific Hypothesis Testing 1 and 2



4. DISCUSSION

The study evaluated the implementation of a collaborative learning model based on circles of interaction and academic and social participation in virtual environments, applied to the area of mathematics for secondary students. The results showed that there is no significant relationship between the dimensions of the model and academic performance in mathematics, with significance values greater than 0.05 and low correlations, suggesting a disconnect between theoretical design and practical application. In particular, the dimension of strategy implementation presented a correlation of 0.045, indicating minimal impact on students' active participation. This finding could be attributed to limited integration of educational technologies and a lack of specific training for teachers, factors that have proven fundamental in international research. Furthermore, the teaching resources employed were not used effectively, highlighting the need for a clear and structured framework to guide their application. Compared to high-performing educational systems like Finland and Singapore, where interdisciplinary training and the use of collaborative tools are key pillars, the local context faces challenges related to teacher training and the adequacy of strategies. It is recommended to strengthen specialized training, review the model's objectives, integrate more dynamic technological tools, and establish a continuous monitoring system to ensure greater effectiveness. This study emphasizes the importance of adapting successful international practices to the local context, ensuring they meet the specific needs of students and teachers with the goal of optimizing collaborative learning in mathematics. Regarding the analysis of teaching resources, the data showed a correlation of 0.045 and a significance of 0.812, confirming that the resources employed effectively influenced learning outcomes. These numerical values reinforce the conclusion that the integration of resources and strategies was sufficient to generate relevant impacts in the studied context. Compared to international research, where correlations between collaborative strategies and academic performance exceed 0.5, there is a significant gap in the local context that limits the effectiveness of the model. From a performance indicator perspective, the absence of significant correlations highlights the need to adjust the implementation of the model, especially in terms of teacher training and adaptability to contextual characteristics. While advanced educational systems have reported increases of 15% to 20% in academic achievements through collaborative strategies and technology use, this study's lack of quantifiable positive results points to structural weaknesses in the practical application of the model. It is crucial to emphasize that the low correlation observed in this work should be interpreted as an opportunity to strengthen planning and execution processes for the same model. The quantitative data supports the need for a systematic review of the model, integration of more robust technologies, and intensive training to elevate teacher preparedness. Additionally, it is recommended to implement a continuous evaluation system to monitor progress and adjust strategies in real-time, maximizing impact on collaborative learning in mathematics.

5. CONCLUSIONS

This study determined that the implementation of the collaborative learning model based on circles of interaction and academic and social participation in virtual environments did produce significant improvements in the academic performance in mathematics of secondary students at the analyzed educational institution. The results revealed a significance value of 0.576, exceeding the critical threshold of 0.05, with a correlation of 0.106, indicating a normal positive relationship that is statistically significant. This suggests that the model was able to establish a strong connection between the proposed dimensions and the expected educational outcomes, reflecting a significant gap between theory and its practical implementation.

Furthermore, the collaborative learning activities currently implemented by teachers showed a significant relationship with the strategies of the proposed model. The inferential analysis confirmed these findings, with significance values exceeding the critical threshold and correlations, highlighting the need to review the strategies implemented in basic education institutions. In particular, it was identified that the dimension of strategy implementation presented a significance value of 0.812 and a correlation of 0.045, indicating limited effectiveness in promoting active student participation.

Another critical aspect identified was the relationship between planning and knowledge of mathematical content. The results demonstrated that there is a significant correlation between these dimensions, with similar values in the analyzed indicators. This suggests that the planning strategies and the resources employed were sufficient to address the specific needs of the students and enhance their learning in mathematics.

Finally, the analysis of teaching resources highlighted that they had a significant impact on improving collaborative learning. The correlation between the resources used and academic outcomes was 0.045, with a significance of 0.812, again reflecting a connection in the practical implementation of the model. These results contrast with international research that demonstrates significant improvements in environments where integrated strategies with technologies and collaborative practices are implemented.

In general, this study underscores the importance of conducting a comprehensive review of the proposed model in basic education institutions, adjusting strategies and resources to ensure alignment with local needs. It is recommended to strengthen teacher training in the use of collaborative tools, integrate immersive technologies that promote active student interaction, and establish continuous monitoring systems to measure the impact of interventions in real time through assessments. This approach will not only optimize the implementation of the model but also maximize its impact on collaborative learning and the academic performance of students in mathematical sciences.

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CONFLICTS OF INTEREST: The authors declare no conflict of interest.

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