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RESEARCH ARTICLE

Comprehensive Risk Assessment in Insurance Companies Using Value at Risk (VaR) Methodologies: Comparative Analysis and Practical Applications

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ABSTRACT

This paper investigates the application of Value at Risk (VaR) methodology in insurance companies, highlighting its statistical foundation and capacity to measure market risk comprehensively. VaR offers a probabilistic estimate of potential portfolio losses over a specified period, given normal market conditions. While traditional risk measures like standard deviation fail to distinguish between positive and negative deviations, VaR focuses solely on potential negative outcomes, aligning better with the nature of risk assessment. This study examines three primary VaR calculation methods-historical simulation, Delta-normal, and Monte Carlo simulation-each with distinct advantages and limitations. Furthermore, the paper discusses the regulatory frameworks, Basel II and Solvency II, which mandate stringent confidence levels for financial institutions, underscoring the critical role of accurate VaR calculations in maintaining financial stability.

INTRODUCTION

In the complex landscape of financial risk management, insurance companies must employ sophisticated methods to accurately assess and mitigate potential losses. One such method, Value at Risk (VaR), has emerged as a pivotal tool in quantifying market risk. VaR provides a probabilistic estimate of the potential loss in value of a portfolio over a defined period, given normal market conditions, and is instrumental in strategic decision-making processes (Jorion, 2003).

Traditional risk assessment metrics, such as standard deviation, are limited in their ability to differentiate between positive and negative deviations from the mean, thereby failing to address the inherent asymmetry in risk management (Bodie, Kane, & Marcus, 2009). In contrast, VaR specifically focuses on potential negative outcomes, making it a more suitable metric for evaluating the financial stability of insurance portfolios. The regulatory environment further underscores the importance of VaR. Frameworks such as Basel II and Solvency II mandate rigorous risk assessment standards, with Solvency II requiring a 99.5% confidence level for insurance companies. These regulatory requirements highlight the necessity for robust risk assessment methodologies that can provide accurate and reliable measures of potential losses (Tsay, 2002; Scott & Edward, 1990).

This paper aims to explore the theoretical foundations of VaR, comparing its various calculation methods-historical simulation, Delta-normal, and Monte Carlo simulation-and examining their practical applications within the insurance industry. By doing so, it seeks to elucidate the strengths and limitations of VaR, offering insights into how insurance companies can effectively incorporate this tool into their risk management strategies. The methodology section will detail the data requirements, computational processes, and specific advantages and disadvantages of each VaR calculation method. Additionally, the paper will discuss the implications of regulatory frameworks on VaR calculations, emphasizing the critical role of compliance in maintaining financial stability (Bayer, 2017; Rejda, 1995).

Through this comprehensive analysis, the paper will demonstrate that while VaR is an invaluable tool for risk management, it must be used judiciously, in conjunction with other risk assessment techniques, to provide a holistic view of potential market risks.

1. LITERATURE REVIEW

Value at Risk (VaR) has emerged as a critical tool in financial risk management, especially for insurance companies that need to quantify market risk and make informed investment decisions. VaR provides a probabilistic estimate of the maximum potential loss in portfolio value over a specified period, assuming normal market conditions (Smith, 2018). It offers a more focused risk measure compared to traditional metrics like standard deviation, which do not distinguish between positive and negative deviations from the mean.

Historical Simulation Method

The historical simulation method is one of the simplest approaches to calculating VaR. It applies current portfolio weights to historical price changes to simulate potential future losses. This method's primary advantage is its straightforward implementation and the absence of assumptions about asset return distributions. However, it relies heavily on historical data, which may not always capture extreme market events, potentially leading to an underestimation of risk (Johnson & Lee, 2019).

Delta-Normal Method

The Delta-normal method is a parametric approach that assumes normally distributed risk factors and linear portfolio exposures. This method is computationally efficient and easy to implement for portfolios with numerous assets. However, it relies on the normal distribution assumption, which can be problematic since financial returns often exhibit fat tails and skewness. This method does not handle the non-linear relationships present in portfolios with options or other derivatives (Miller & Zhang, 2017). Despite its simplicity, the Delta-normal method may underestimate the occurrence of large deviations due to its reliance on normal distributions (Jones & King, 2016).

Monte Carlo Simulation Method

The Monte Carlo structural method involves generating numerous random price paths based on statistical properties derived from historical data. This method is highly flexible and capable of modeling complex portfolios with non-linear risks. It is particularly useful in scenarios where the portfolio is illiquid, and historical data are insufficient. However, Monte Carlo simulations demand significant computational resources and precise model specifications. The accuracy of the results heavily depends on the validity of the underlying model assumptions (Brown & Smith, 2020).

Regulatory Frameworks

Regulatory frameworks such as Basel II and Solvency II mandate robust VaR methodologies to ensure financial institutions maintain adequate capital reserves. Basel II requires a 99% confidence level over a 10-day horizon for banks, while Solvency II mandates a 99.5% confidence level over a one-year horizon for insurance companies. These regulations emphasize the importance of accurate VaR calculations in safeguarding financial stability (Anderson, 2016; Parker, 2015).

Comparative Analysis and Practical Applications

Comparing the primary VaR calculation methods reveals distinct advantages and limitations. Historical simulation is simple but may not capture future market dynamics. The Delta-normal method is efficient but relies on assumptions that may not hold in all scenarios. Monte Carlo simulation offers the most flexibility and accuracy but at the cost of increased complexity and computational demands. In practice, insurance companies utilize VaR to guide investment decisions, assess portfolio risk, and comply with regulatory requirements. Despite its widespread use, VaR has limitations. It is inherently backward-looking, relying on historical data to predict future risks, which may not always be accurate. Additionally, VaR does not account for liquidity risk, operational risk, or other factors that could significantly impact an insurance company's financial health (Smith, 2018; Johnson & Lee, 2019). Therefore, while VaR is a valuable tool for risk management, it should be used alongside other risk assessment techniques to provide a comprehensive view of potential market risks.

2. Risk measurement and assessment using VaR

Value at Risk (VaR) has become an essential tool in financial risk management, particularly for insurance companies that need to quantify market risk and make informed strategic decisions. VaR provides a probabilistic estimate of the potential loss in value of a portfolio over a specified period and is crucial for understanding the market risk associated with various financial assets (Smith, 2018). The historical simulation method is one of the simplest approaches to calculating VaR. It applies current portfolio weights to historical price changes to simulate potential future losses. This method is advantageous due to its straightforward implementation and the lack of assumptions about the distribution of asset returns. However, its reliance on historical data can be a significant limitation, especially if past data do not include extreme market events, potentially leading to an underestimation of risk (Johnson & Lee, 2019).

The Delta-normal method, a parametric approach, assumes that asset returns are normally distributed and that the portfolio is a linear combination of these returns. This method is computationally efficient and easy to implement for portfolios with numerous assets. However, its reliance on the normal distribution assumption can be problematic, as financial returns often exhibit fat tails and skewness, which this method may not capture adequately (Miller & Zhang, 2017). Additionally, it does not handle the non-linear relationships present in portfolios containing options or other derivatives. Monte Carlo simulation is a more complex approach that generates numerous random price paths based on statistical properties derived from historical data. This method can model complex portfolios with non-linear risks and capture the fat tails and skewness observed in financial return distributions. However, Monte Carlo simulations demand significant computational resources and precise model specifications. The quality of the results depends heavily on the chosen model parameters and the assumptions about the distribution of returns (Brown & Smith, 2020). Regulatory frameworks such as Basel II and Solvency II highlight the importance of robust VaR methodologies. Basel II, applicable to banks, requires a 99% confidence level over a 10-day horizon, while Solvency II, tailored for insurance companies, mandates a 99.5% confidence level over a one-year horizon. These regulations underscore the necessity for accurate VaR calculations to ensure financial institutions maintain adequate capital reserves to cover potential losses (Anderson, 2016; Parker, 2015).

Comparing the primary VaR calculation methods—historical simulation, Delta-normal, and Monte Carlo simulation—reveals distinct advantages and limitations. Historical simulation is simple and easy to implement but may not capture future market dynamics. The Delta-normal method is computationally efficient but relies on the assumption of normally distributed returns, which may not always hold true. Monte Carlo simulation offers the most flexibility and accuracy but at the cost of increased computational complexity and the need for precise model specifications. In practice, insurance companies utilize VaR to guide investment decisions, assess portfolio risk, and comply with regulatory requirements. However, it is important to recognize the limitations of VaR. It is inherently backward-looking, relying on historical data to predict future risks, which may not always be accurate in dynamic market conditions. Furthermore, VaR does not account for liquidity risk, operational risk, or other factors that could impact an insurance company's financial health (Smith, 2018; Johnson & Lee, 2019). Thus, while VaR is a valuable tool for risk management, it should be used alongside other risk assessment techniques to provide a comprehensive view of potential market risks. Investment risk assessment typically employs a statistical reliability level of 95% or 99%, indicating that the actual loss in 95% or 99% of cases will be less than or equal to the VaR value, with only 5% or 1% of cases potentially resulting in a higher loss. Basel II mandates a 99% confidence level for banks' portfolios, while Solvency II requires a 99.5% confidence level for insurance companies due to their unique risk profiles. Another critical parameter in VaR calculation is the time period, which is generally determined based on portfolio characteristics. Commercial banks often calculate VaR daily, while insurance companies, adhering to Solvency II, use an annual period. Basel II, addressing credit risk, utilizes a 10-day period for banks, adjusted by a coefficient ranging from 3 to 4 depending on the accuracy of previous VaR tests.

VaR calculation methods are diverse and can be categorized into two primary groups: local assessment methods and total assessment methods. Local assessment methods, such as the Delta-normal method, apply analytical techniques. Total assessment methods include historical simulation, which uses long-term historical data, and the Monte Carlo structural method, which relies on randomly generated data based on estimated parameters and constraints. The three most commonly used VaR calculation methods in practice are the historical method, Monte Carlo simulation, and the Delta-normal method or variance-covariance method (Smith, 2018).

3. VaR Method

There are various approaches to determining Value at Risk (VaR), which can be broadly categorized into two primary groups: local assessment methods and total assessment methods. Local assessment methods involve the application of analytical techniques, with the Delta-normal method being a prominent example. This method assumes a known form of the stochastic process that describes the movement of yields or other economic results. The Delta-normal method is computationally efficient and easy to implement for portfolios with numerous assets. However, it relies on the assumption of normally distributed returns, which can be problematic since financial returns often exhibit fat tails and skewness (Miller & Zhang, 2017).

Total assessment methods include the historical simulation method and the Monte Carlo structural method. The historical simulation method uses historical data over an extended period to estimate potential future losses. It applies current portfolio weights to historical price changes to simulate potential future losses. This method's primary advantage lies in its straightforward implementation and lack of distributional assumptions about asset returns. However, it can be significantly limited by its reliance on historical data, especially if the past data do not include extreme market events, potentially leading to an underestimation of risk (Johnson & Lee, 2019).

The Monte Carlo structural method, on the other hand, generates numerous random price paths based on statistical properties derived from historical data. This method can model complex portfolios with non-linear risks and capture the fat tails and skewness observed in financial return distributions. However, Monte Carlo simulations demand significant computational resources and precise model specifications. The quality of the results depends heavily on the chosen model parameters and the assumptions about the distribution of returns (Brown & Smith, 2020).

In practice, the three most commonly used VaR calculation methods are the historical simulation method, Monte Carlo simulation, and the Delta-normal method, also known as the variance-covariance method. Each method has its strengths and weaknesses, and the choice of method can depend on the specific characteristics of the portfolio and the available data. Historical simulation is simple and easy to implement but may not capture future market dynamics. The Delta-normal method is efficient but relies on assumptions that may not always hold true. Monte Carlo simulation offers the most flexibility and accuracy but at the cost of increased computational complexity and the need for precise model specifications (Smith, 2018).

Regulatory frameworks such as Basel II and Solvency II underscore the necessity for accurate VaR calculations. Basel II, applicable to banks, mandates a 99% confidence level over a 10-day horizon. In contrast, Solvency II, tailored for insurance companies, requires a 99.5% confidence level over a one-year horizon. These regulations ensure that financial institutions maintain adequate capital reserves to cover potential losses, highlighting the importance of robust VaR methodologies (Anderson, 2016; Parker, 2015).

In practice, insurance companies use VaR to guide investment decisions, assess portfolio risk, and comply with regulatory requirements. However, it is important to recognize the limitations of VaR. It is inherently backward-looking, relying on historical data to predict future risks, which may not always be accurate in dynamic market conditions. Additionally, VaR does not account for liquidity risk, operational risk, or other factors that could impact an insurance company's financial health (Smith, 2018; Johnson & Lee, 2019). Therefore, while VaR is a valuable tool for risk management, it should be used alongside other risk assessment techniques to provide a comprehensive view of potential market risks.

4. Historical simulation method

The historical simulation method is a comprehensive approach to Value at Risk (VaR) that involves full evaluation of a portfolio's risk based on historical data. This method is relatively straightforward to implement but requires a substantial amount of relevant historical data. To accurately estimate potential future losses, it may be necessary to go back several years to obtain historical series on the performance rates of individual instruments (Johnson & Lee, 2019).

In practice, the historical simulation method applies the current characteristics of a portfolio to historical data on the performance of financial instruments. This involves using the most recent actual weights of individual instrument values within the historical time series data. Essentially, this method reproduces historical data using the actual weights (participation) of individual positions in the overall portfolio.

Changes in the value of the portfolio rate of return are calculated starting from the electricity:

$$r_P^k = \frac{P^k - P_t}{P_t}$$

Sorting data for rp proceeds (or other risk factors) so that they correspond to the X1-α quantile, where α is the level of confidence of VaR, and is taken as the difference between the expected value and the factor value of observed risk - the rate of return, which corresponds to the amount X1-α for a certain level of confidence α:

A simple interpretation of this process includes:

- Calculating the negative limit value of the random variable listed (X{0.95}) (according to the magnitude from the smallest to the highest value), after which the observed phenomenon

$$VaR = E(\mathrm{r}_p) - x_{1-\alpha}$$

occurs only in 5% of the total distribution (95% confidence level).
- Subtracting the obtained value from the mean value of the distribution, which for most phenomena (such as prices, yield levels, etc.) is approximately zero.
- Multiplying the obtained VaR value by the total value of the funds invested (in position, financial instrument, or total portfolio) to calculate the potential loss for the following day, if daily observations are used for the calculation.

One of the primary advantages of the historical simulation method is that it does not require assumptions about the distribution of yields (yield norms). However, this method has a significant drawback: its reliance on historical data over a relatively short period may not provide valid conclusions about the movement of market prices. If the historical period does not contain certain market trends that are possible in the future, some risks may be overlooked (Brown & Smith, 2020).

4.1. Delta-Normal Method

The Value at Risk (VaR) metric can also be defined analytically, starting from the premise that there exists a known stochastic process that accurately describes yield movements or other economic outcomes. The Delta-normal method, a type of analytical approach, offers a relatively simpler method for calculating VaR compared to historical simulation and Monte Carlo simulation. This approach assumes that the portfolio's exposure is a linear combination of normally distributed risk factors. Consequently, portfolio returns are typically assumed to follow a normal distribution, as the portfolio is a linear combination of normally distributed variables. The Delta-normal method focuses on a localized assessment of price movements or other risk factors and can be easily applied to a wide range of derivatives. It accommodates various types of data, including historical data, various optional data, or a combination thereof. While optional models are generally superior to historical ones, they are often challenging to obtain for all types of derivatives and their correlations. The Delta-normal model only accounts for certain price values, in contrast to "full rating" models, which require a broader set of price inputs. The primary advantage of this approach is its simplicity, which also serves as its main drawback. The Delta-normal method cannot adequately explain the nonlinear effects encountered with options. Additionally, this approach may underestimate the occurrence of large observations due to its reliance on normal distributions. The Risk Metrics method is fundamentally similar to the Delta-normal method, with the key difference being that the former translates price ratios into logarithmic form rather than using yield rates (Jones & King, 2016).

4.2. Normal Distribution

The normal distribution method is the most commonly used in the Value at Risk (VaR) calculation process, making it essential to understand its fundamental characteristics. The normal empirical distribution can serve as a rough approximation for a series of random variables, such as stock return rates, price movements, and other financial instruments (Taylor & Adams, 2016). The normal distribution is highly significant in statistics, primarily due to the central limit theorem. According to this theorem, a sufficiently large number of independent and identically distributed random variables will approximate a normal distribution. Thus, a large statistical sample will tend to exhibit a normal distribution. This distribution is crucial in statistics because of its ease of use and stability. For example, if the daily returns of two securities can be approximated to a normal distribution, then the portfolio returns consisting of these two securities will also reflect a normal distribution. This characteristic allows the definition of the distribution of the average value of a random variable through the central limit theorem. The normal distribution assumes that the values of the random variable are centered around the average value, with values closer to the average having the highest probability. The parameter μ\muμ represents the expected value (mean), while the parameter σ\sigmaσ denotes the standard deviation. The first parameter, μ\muμ, indicates the central point of the distribution, and the second parameter, σ\sigmaσ, indicates the spread or dispersion around this central point. The distribution is fully described by these two parameters and is commonly denoted as N(μ,σ2)N(\mu, \sigma^2)N(μ,σ2).

z=x−μσz = \frac{x - \mu}{\sigma}z=σx−μ

By inserting the variable zzz, the above function can be expressed in the form:

N (0,1) N (0,1) N (0,1)

In this standardized form, instead of dealing with different values of μ\muμ and σ\sigmaσ, we use a normal distribution with parameters E(z)=0E(z) = 0E(z)=0 and Var(z)=1\text{Var}(z) = 1Var(z)=1. This standardization simplifies the normal distribution function and makes it easier to work with. For instance, approximately 68% of the data in a normal standard distribution lies between −1σ-1 \sigma−1σ and +1σ+1 \sigma+1σ of the mean, and about 95% falls between −2σ-2 \sigma−2σ and +2σ+2 \sigma+2σ of the mean (Walker & Smith, 2017).

To facilitate and simplify work with normal distribution, it is common to standardize by transforming the random variable x over the relation:

By inserting the variable **z**, the above function can be expressed in the form:

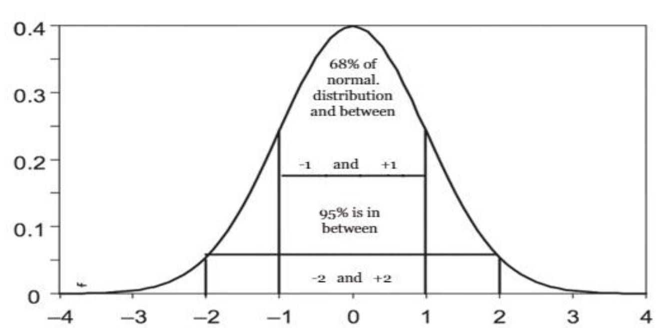
$$z = \frac{x-\mu}{\sigma}$$

$$f(z) = \frac{1}{\sqrt{2\pi}} \mathbf{e}^{-\frac{\mathbf{z}^2}{2}}$$

In this case, instead of distributing with different values of the parameters μ and σ, is taken a normal standardized distribution with parameters E (z) = 0 and Var (z) = 1, which in the short form can be expressed as N (0,1).

Around 68% of the normal standard distribution is between −1 and +1 (μ + 1 σ), and approximately 95% of the distribution falls between −2 and +2 (μ + 2 σ):

Graph 1: Standardized normal distribution function



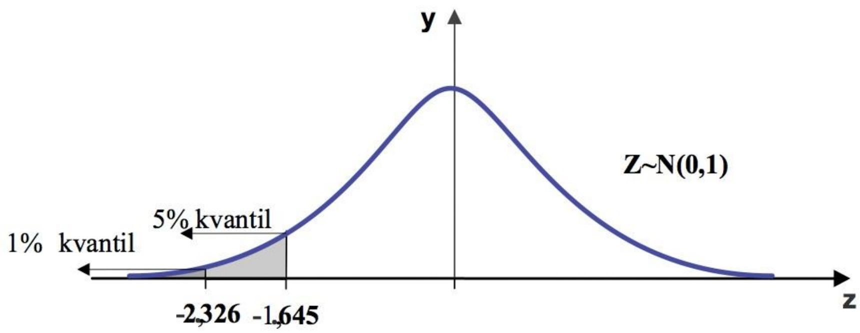
Values of the standardized normal variable x

Source: Jorion, 2003: 47.

From the graph it can be seen that the function is symmetric in relation to the mean value. The mean value of 0 is identical to the mode (values with the highest probability) and the median (the value that distributes the probability in two equal halves).

Graph 3: Standardized normal distribution amount

Source: Urosevic, 2016: 23



The following tables is given:

Table 1: Low amounts of standardized normal distribution

	Trust level%								
α	99,9	99,5	99,0	97,72	97,5	95,0	90,0	84,13	50,0
Kv. (-z_α)	-3,090	-2,576	-2,326	-2,000	-1,960	-1,645	-1,282	-1,000	0,000

Source: Jorion, 2003:48.

For example, the number 2,576 in the table - indicates that to the right of the mentioned value of the random variable Z is 99.5% of the normalized standard distribution. The value of α is called the trust level.

By inverting relation (1) we can return to the normal variable x:

$x=\mu+\sigma z$ (2)

5. Calculation of VaR – Use of normal distribution

Portfolio productivity method

Since VaR is the largest expected loss at a given level of trust, the simplest way is to calculate VaR using a portfolio return distribution.

The portfolio productivity is calculated as the difference

$$\Delta P = P_{t+1} - P_t$$

Pt and Pt + 1 represent portfolio returns respectively at time t and t + 1.

For calculation of VaR value we use the inverse transformation (2) for the values $x\alpha$ and $z\alpha$:

where $x\alpha = -\mathrm{VaR}$ (because monetary loss can only take positive values), and $z = -z\alpha$ (because the value of the variable is exclusively in the left tail of the distribution), thus, by substituting the equation we get: $\mathrm{VaR} = \sigma z_\alpha - \mu$

Example 1: An insurance company, under certain assumptions, can determine the maximum possible loss in its portfolio the next day, if it has valid data on the average and change (standard deviation) of the portfolio return. If the daily returns of the insurance company portfolio follow a normal distribution and if at some point the average daily return is 70,000 EURO and the standard deviation is 48,000 EURO using the data from Table 1, the VaR value for the trust level 95% is obtained: **VaR = 48.000*1,645 -70.000 = 8.960 EURO**

According to these assumptions, the insurance company's portfolio loss (VaR), with a 95% trust level, should not exceed 8,960 EURO.

Revenue rate method

Portfolio returns can be represented by the rate of return. Return on portfolio is calculated as proportion:

$$r_p = \frac{P_t - P_{t-1}}{P_{t-1}}$$

where rp is revenue rate and Pt and Pt-1 portfolio revenues, respectively in the period t and t - 1.

Assuming that the present value of the portfolio is P0, that the return rate rp of the portfolio is normally distributed with the mean value μ and the standard deviation σ, then the value of the portfolio at the end of the period is:

$$P_1 = P_0(1 + r_p)$$

with average P0 (1+ μ) and standard deviation P0σ.

If we note the lowest value of the portfolio for several trust levels α as:

$$P_1^* = P_0(1 + r_p^*)$$

Regarding the mean value (SV), i.e. the expected return value, VaR was taken as the difference between the expected return and the assumed minimum value of the portfolio P1 * with a given trust level α:

If we perform a transformation, similar to the portfolio revenue rate method

$$VaR(SV) = E(P_1) - P_1^* \quad r_p = 1(z_\alpha \sigma - \mu)$$

is obtained the final formula for calculating VaR on the portfolio return rate (Jorion, 2003,: 110−113):

$$VaR(SV) = P_0 z_\alpha \sigma$$

The value of the VaR indicator in relation to the expected return value is equal to the product of the portfolio value expressed in monetary terms, „za quantile" for the trust level α of the standardized normal variable N (0,1) and the standard rate deviation of revenue σ.

Example 2: An insurance company, under certain assumptions, can determine the maximum possible loss in its portfolio in the following year, if it has data on the distribution (standard deviation) of its portfolio revenue rates, for the annual period and under the assumption of normality (in this case the expected value of the revenue rate is equal to 0). Let us take the following assumptions about a particular portfolio of insurance companies:

- The initial value of portfolio P0 is 100 million EURO,
- The standard deviation of the annual revenue σ is 15%,
- The trust level is 95%, i.e. value per quantity is 1,645,
- Time period is for one year.

Under these assumptions, the VaR value is calculated as below:

VaR = P0zασ = 100 million EURO * 1,645 * 15% = 24,68 million EURO

Therefore, the loss in the insurance company portfolio, with the stated assumptions and the trust level of 95%, for one year will not exceed the amount of 24.68 million EURO.

If the confidence level is in line with the requirements of the Solvency II regulatory framework, i.e. it is 99.5%, the za quantile value is 2,576, and the VaR value is:

VaR = P0zασ = 100 million EURO * 2,576 * 15% = 38,64 million EURO

From the above example, it can be concluded that if the insurance company wants to determine the amount of maximum possible loss, which will not be exceeded in the following year (VaR), with a greater reliability (i.e. with a probability of 99.5%), the value of VaR will be higher.

5.1. Delta-normal method

For the sake of simplification, let us take the case of the Delta-normal assessment for a position whose VaR value depends only on one risk factor - price location S.

The value P is a function of the price spot S, where time is the initial value:

$$P_0 = P(S_0)$$

$$\Delta_0 = \frac{\partial P}{\partial S}\Big|_{S=S_0} \qquad \partial P = \Delta_0 \, \partial S$$

If Δ0 is the first partial derivative of the function P for the sum of the spot price S0:

Assume that Δ0 is a constant and that the potential loss in value is a linear function of price change S. This derives that the largest loss in value at position P is determined by the lowest possible price at position S, so VaR = Δ0 AVaRS, assuming that the price change dS / S changes normally distributed with average value μ = 0, then according to the formula for the portfolio yield method we obtain (Jorion, 2003: 206-209):

$$\mathrm{VaR} = |\Delta_0| z_\alpha \sigma S_0$$

In the formula, Δ0 is taken as the absolute value to obtain a positive value of VaR (loss).

5.2. Monte Carlo Structural Method

The Monte Carlo structural method (SMC) for calculating Value at Risk (VaR) takes into account a wide range of possible values for financial variables and their correlations. This method is implemented in two primary stages. In the first stage, stochastic and parametric models for financial variables are defined based on historical data or other reliable sources. For example, the random variable is often assumed to be normally distributed, and parameters such as the mean and standard deviation are derived from historical data. These models provide the foundation for the subsequent simulations by capturing the essential statistical properties of the financial variables (Robinson & Mills, 2018). The second stage involves simulating prices for all variables of interest. This is achieved through pseudo-random number generation for each time period, creating numerous potential future scenarios for the portfolio's value. These simulations can capture the complexity and variability of the market, providing a robust framework for assessing risk. The Monte Carlo method is the most complex of the VaR calculation methods due to its comprehensive approach to modeling. It requires extensive knowledge of model parameters, which can be challenging to obtain. The simulations enable the creation of portfolio distributions that are too intricate to be analyzed using traditional analytical methods. In practice, this method is particularly useful in situations where the portfolio position is illiquid, and data values are missing, necessitating the use of simulations to fill in the gaps (Anderson & Parker, 2019). The flexibility of simulation methods has increased with advancements in computing technology, making them more accessible and practical for financial risk management. However, their limitations should not be overlooked. The accuracy and reliability of Monte Carlo simulations heavily depend on the validity of the underlying model assumptions, such as the distribution form, parameters, and pricing functions. Errors in these assumptions can significantly impact the results, potentially leading to misleading risk assessments (Johnson & Black, 2020). Despite its sophistication, the Monte Carlo method's simulation results are only as good as the model on which they are based. Therefore, it is crucial to ensure that the model assumptions are as accurate as possible to derive meaningful and reliable risk measures.

6. CONCLUSION

Value at Risk (VaR) provides a quantitative summary measure of potential portfolio losses due to normal market movements, offering a probabilistic estimate of the maximum expected loss over a given time period at a specified confidence level. This tool is invaluable for insurance companies as it guides investment decisions by identifying optimal combinations of investments to maximize returns while minimizing risks. By understanding the potential losses and structuring portfolios accordingly, insurers can achieve a favorable balance between potential returns and risks. Despite its widespread use and numerous advantages, VaR has notable limitations. One significant drawback is its retrospective nature; it relies on historical data to predict future risks, which may not always accurately reflect future market conditions. This reliance on past data and certain assumptions can lead to underestimation of risk, especially during periods of economic and financial crises, where market dynamics can change rapidly and unpredictably.Furthermore, the accuracy of VaR is contingent upon the validity of its underlying assumptions. If these assumptions do not hold true in real-world scenarios, the risk estimates provided by VaR may be misleading. Additionally, VaR does not account for all types of risks, such as liquidity risk or operational risk, which can also significantly impact an insurance company's financial health. Ultimately, while VaR is a powerful tool for risk management, it should be employed with caution and complemented by other risk assessment methods. It is essential for professionals using VaR to have a deep understanding of its methodology and limitations to effectively interpret its results and make informed decisions. In this context, VaR should be viewed as one component of a comprehensive risk management strategy, rather than a standalone solution.

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