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RESEARCH ARTICLE

The Effect of Active Learning on Mathematical Representations among Secondary School Students

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ABSTRACT

The goal of the research is to identify the effect of active learning on mathematical representations among secondary school students. To achieve the goal of the research, the two researchers adopted a quasi-experimental research design with two groups (experimental and control) with a post-test, based on this a following null hypothesis was developed: There is no statistically significant difference at the significance level (0.05) between the average scores of the students in the experimental group who studied according to active learning and the scores of the students in the control group who studied in the usual way in the mathematical representations test. The research sample consisted of (66) students from the fifth scientific grade in one of the schools affiliated with the General Directorate of Education of Baghdad / Al-Karkh II, and the sample was distributed into two groups: 1. The experimental group consisted of (34) students who studied according to active learning. 2. The control group consisted of (32) students who studied in the usual way. Equivalence was conducted between the two groups in the variables (previous achievement in mathematics, previous mathematical knowledge, intelligence, mathematical representations), and the research tool represented by the mathematical representations test was built. The test consisted of (36) essay and substantive items, appropriate statistical analyzes were conducted, and the psychometric properties of the test were confirmed. The outcomes resulted in the students of the experimental group outperforming the students of the control group. In light of these results, a number of conclusions, recommendations, and proposals were reached.

INTRODUCTION

The reality in schools indicates that the most difficult problems facing middle school students in their study of mathematics is their transition from semi-tangible to abstract symbols, and this constitutes an obstacle for them, Thus, we find that their learning stops at the level of memorization and retrieval, and what helped them in this is the traditional teaching methods used by teachers, and their lack of familiarity with modern strategies in teaching mathematics, Therefore, the National Council of Teachers of Mathematics called for the principle of learning and set principles and standards for it. Mathematical representation is one of the important standards called for by the Council, It is like the heart of the body in learning and teaching mathematics, and the most important main goals of the mathematics curriculum. The two researchers noticed, by analyzing the mathematics textbook for the fifth scientific grade for the components and indicators of mathematical representations, its focus on two components: verbal language and written symbols, and the appearance of the rest of the components in small percentages (verbal language 36% Written symbols 37%, shapes and pictures 1%, models and models 1%, life situations 4%, graph tables 5%, graphs 7%, mathematical equations 9%., This is contrary to what (NCTM, 2000, p16) emphasized the necessity of building curricula in light of the components of mathematical representations, and from the experience of the two researchers in the field of teaching, it was noted that there is still a weakness among teachers in employing mathematical representations and their application of specific types of them, In addition, teachers are concerned about the difficulty of students mastering mathematical representation, Which requires them to put in more time and effort to train them, and this was confirmed by the results of the mathematical representations test that the two researchers conducted for the research sample for the purposes of equivalence, which includes questions that measure some of its components. The results showed that the average test score was (3.487) while the total score was (10) with a success rate of (29%) and a standard deviation of (1.424). This is an indication of the weak level of mathematical representation among students. Despite the importance of mathematical representations, references and reports of education directorates showed a decline in students' mathematical representations. According to this, what is observed in reality of the deficiency in mathematical representation processes among secondaryschool students is that this weakness is due to the use of ineffective and traditional teaching methods, and without providing opportunities for students to participate, dialogue, and communicate about this information, so it was necessary to adopt a teaching strategy that encourages active learning. It makes the learner the focus of the teaching-learning process, and therefore the research problem can be determined by answering the following question:

What is the effect of active learning on mathematical representations among middle school students?

Research importance

The importance of the research can be demonstrated by the following:

1. Mathematics is one of the most important subjects taught, and a basic subject in daily life, in scientific communication, and in developing the ability to think and confront problems, and it has a role in the scientific and technological explosion that the world is experiencing.
2. This research targets secondary school students, which has its own characteristics: Because it is located in the middle between middle school and university education, which requires taking into account the nature of the learners and providing them with effective education and sustainable higher thinking skills that do not depend on memorization and memorization.
3. It may encourage teachers to develop their teaching methods using active learning strategies.
4. Providing an educational environment based on active learning to develop mathematical representations.
5. Officials in curriculum development may draw their attention to the need to increase the use of mathematical representations, include them in the school curricula, and train teachers on them.
6. The research presented a test of mathematical representations for fifth scientific-grade students that researchers, teachers, and those interested in teaching mathematics can benefit from.

Research objective

The current research aims to determine the effect of active learning on mathematical representations among secondary school students.

Research limits

1. Students of the fifth scientific grade in secondary day and government schools affiliated with the General Directorate of Education of Baghdad / Al-Karkh II for the year 2023-2024.
2. Content of the first four chapters of the mathematics textbook for the fifth scientific grade, 12th edition, 2023 (logarithms, sequences, conic sections, circular functions).
3. The first semester of the academic year 2023/2024
4. Proposed strategic steps according to active learning: (preparation and integration of information, modeling and construction of meaning, application of ideas, feedback)...
5. Components of the proposed mathematical representations model: (verbal language, written symbols, pictures and shapes, models and models, life situations, graphs, charts, mathematical equations).

Terms definitions

Impact: Defined by (Shehata and Al-Najjar, 2003) as: "the result of a desired or undesirable change that occurs in the student as a result of the learning process." (Shehata and Al-Najjar, 2003, p. 22)

The two researchers define it procedurally as the amount of change that active learning brings about in the mathematical representations of the experimental group students in mathematics, and it can be observed and measured in the average grades of the fifth scientific grade students after completing the experiment.

Strategy: Defined by (Al-Kubaisi, 2015) as: "A group of movements carried out by the teacher in the classroom, which occur in an organized and sequential manner aiming to achieve the teaching objectives prepared in advance." (Al-Kubaisi, 2015, p. 24)

The two researchers define it procedurally as a set of planned steps, procedures, and activities that are implemented sequentially according to active learning, starting with (preparation and integration of information, modeling and construction of meaning, application of ideas, and feedback) for teaching mathematics to fifth scientific -grade students (experimental group).

Active learning: Defined by (Saada et al., 2011) as: "a method of teaching and learning at the same time, whereby students participate in projects, activities, and exercises very actively, in a diverse and rich educational environment that allows them rich discussion, positive listening, constructive dialogue, and conscious thinking and analysis and deep contemplation of everything that is presented, read, or written in academic subjects, under the supervision of a teacher who encourages them to achieve ambitious goals." (Saada et al., 2011, p. 33)

The two researchers define it procedurally as learning that makes the fifth scientific grade student the focus of the educational process, and makes him positive, active, and effective in the educational process through the proposed active learning strategy. He seeks and searches to obtain information, and carries out activities to apply what he has learned, as the teacher's role in it is limited to direction Encouragement and guidance.

Mathematical representations: Defined by Badawi (2007) as: "expressing the mathematical idea using various means such as words, pictures, illustrations, graphs, tables, and mathematical relationships, and this representation helps them understand the concepts." (Badawi, 2007, p. 59)

The two researchers define it procedurally as the ability of fifth scientific-grade students from (the research sample) to translate mathematical ideas and concepts using the components of the proposed mathematical representations model (verbal language, written symbols, pictures and shapes, models and models, life situations, spreadsheets, graphs, mathematical equations). To other equivalent forms to understand and translate the mathematical idea or to reach correct solutions to the mathematical problem.

THEORETICAL FRAMEWORK

Constructivist theory

Constructivist theory is one of the theories of modern learning that has grown and expanded in its fields until it has become a theory of learning and teaching. Constructivism became the dominant force in education in the nineties of the last century, and its perspective represents a synthesis of a number of ideas drawn from three fields: psychology. Cognitive, psychology and anthropology (Zaytoun, 2008, pp. 32-49), One of the most prominent theorists of constructivist theory is Jean Piaget, who believes that intelligence is a form of advanced adaptation that develops quickly through the processes of representation and adaptation and does not appear suddenly (Piaget, 2004, p. 28). Constructivism is based on a set of principles and assumptions that reflect its features, It is represented by the fact that previous knowledge is necessary for new learning to occur, and that learning is a constructive, purpose-oriented, active, and continuous process, as the student builds his knowledge through his social interaction with others, and learning occurs when the student faces a real problem or task (Al-Saadi et al., 2021, p. 147) .

Active learning

What is active learning?

Active learning appeared in the late twentieth century, through the report sent by Bonwell & Eison in 1991 to the American Association for Higher Education. This is due to the dominance of traditional lectures in college classrooms. The report included an explanation of the nature of active learning and experimental research on the application of some of its strategies (modified lecture, discussion and problem solving, peer learning, cooperative learning) and the obstacles that prevent its application. the report urged faculty members in colleges and universities to actively involve students in the learning process. (Bonwell & Eison, 1991, p5). Active learning was not just a set of activities carried out by the learner; But it is a behavior or direction for both the learner and the teacher that makes education effective (Badawi, 2010, p. 162). Teaching for understanding and meaning has a positive impact on students' learning, has many returns, and greater application of ideas (Al-Mayouf et al., 2016, p. 10), Active learning allows students to reflect, speak, write, and listen through the use of multiple methods, such as small groups and role-playing (Ambou Saidi, 2016, p. 25).

Constructivist theory and active learning

The constructivist theory is one of the theories in psychology that has most contributed to defining and crystallizing the philosophy, foundations, principles and characteristics of active learning, and it has also helped in reaching to many of its strategies (Al-Khalili et al., 1996, p. 43). Jaballah (2016, p. 90), citing (Zaytoun, 1998), believes that the assumptions and results of the constructivist theory represent the foundations from which active learning begins. The constructivist theory supports the learner's activity in the educational situation, and the more active the learner is during learning, the more capable he is of building his knowledge. And the formation of a strong, interconnected cognitive structure. Learning according to constructivist theory occurs when the ideas that the learner possesses are modified, or new knowledge and information are added to them, or the ideas that the learner has are reorganized in the light of new knowledge, meaning that the focus here is not only on the educational product, but rather Focus on the processes and environment in which the learning process takes place to obtain knowledge and build new experiences.

Foundations of active learning

Active learning depends on several foundations, the most prominent of which are:

1. Involving students in choosing the work system and rules, and determining their educational goals.
2. Use student-centered teaching strategies and methods.
3. Allow students to ask questions to each other or to the teacher according to a previously agreed-upon mechanism.
4. Taking into account individual differences among students by using various methods to suit all students.
5. Diversify the sources of knowledge and learning, and direct students to them while taking into account freedom of choice.
6. Providing a fun and comfortable educational environment that encourages learning and stimulates thinking, and can be formed in groups.
7. Ensure that learning is related to students' lives and realistic.

(Abu Al-Hajj and Reconciliation, 2016, p. 23)

The importance of active learning and its benefits

Learning is an active process. Students build knowledge as they explore the world around them, reflect, observe and interact with phenomena, make connections between new ideas and previous understandings, and include enriching and changing current understanding. (Abdo, 2022, p. 18), and the importance of active learning becomes clear by encouraging students to work Positive, and helping students acquire knowledge, as (Hassan, 2023, p106) believes that organizing students' learning into groups develops the spirit of cooperation and knowledge exchange, as they build a safe educational environment for discussions, and in which students feel comfortable.

(Badawi, 2010, p. 180) and (Al-Zaidi, 2009, p. 63) mentioned benefits of active learning, including:

1. Through active learning, students reach meaningful solutions when solving problems; because they link the new information they learned to familiar ideas retrieved from their memory.
2. Active learning increases students' motivation to be more active and effective instead of passive.
3. Through active learning, students receive reinforcements in their understanding of new knowledge.
4. Active learning increases students' confidence and self-reliance.
5. Students receive immediate feedback more frequently.
6. The teacher is not the only source of knowledge in active learning.

The role of the teacher and learner in active learning

(Awad and Zamel, 2010, p. 34), (Khairy, 2018, pp. 89-95), (Al-Saadi, 2021, p. 271), and (Saadah et al., 2011, pp. 113-124) summarized the roles of both the teacher and the learner as followes:-

The role of the teacher in active learning

1. The teacher provides the human and material resources that help the student in active learning, including providing the appropriate place and time to facilitate it.
2. Providing assistance to students in discovering new experiences, intervening to correct thinking, and providing them with immediate feedback.
3. Distributing assignments to students, and providing instructions and directions to them.
4. Providing a friendly and supportive climate for learners, maintaining class control and managing discussion and dialogue.
5. Use and design appropriate educational strategies and activities.

The role of the learner in active learning.

1. Searches for different sources to acquire knowledge, and takes responsibility for his learning.
2. Expresses his different ideas and ambitions.
3. Accepts the directions and instructions provided by the teacher with all friendliness and gratitude.
4. Contributes to the evaluation of others and himself.
5. Uses what he has learned in his daily life.

(Jawad, 2022, p424) believes that scientific thinking, acquiring new knowledge, and solving problems can only be achieved by the active, effective learner who acts positively towards the learning process. (Majeed.at el, 2023, p204) adds that the teacher in active learning must be a good role model in his thinking, movements, and directions, and thus be a good role model for his students, guide them toward sound thinking skills, and provide them with the right directions in their behavior to face different life situations.

The two researchers believe that active learning seeks to ensure that students are able to learn new experiences through practice and so that they can apply them in their daily lives, and develop their abilities in various aspects, so that they become useful to themselves, their society, and their future. Through the research experience in this study, the two researchers sought as much as possible to implement the roles of both the teacher and the learner mentioned above, to ensure the achievement of the objectives of the study. They also focused on the relationship between them being based on mutual cooperation to facilitate the maintenance of classroom discipline.

The proposed strategy according to active learning

The teaching strategy includes all procedures, events and activities that contribute to achieving the educational outcomes of knowledge, values, attitudes and skills. Choosing the appropriate strategies depends on a set of criteria, which aim to achieve the lesson objectives, including:

1. Objectives: Teaching strategies are linked to the objectives to be achieved from the lesson.
2. Scientific subject: The nature of the scientific subject, whether theoretical or practical, contributes to determining the appropriate and effective teaching method, and therefore the teaching strategy varies from one subject to another.
3. Material capabilities and resources: This includes equipment and tools, places available for training, and the availability of financial resources.
4. Human resources: They include teachers and technical mentors, their scientific and educational specializations, their competence, as well as learners and their categories, the nature of their development, their number, their previous experiences, and the educational stage they are in.
5. Principles and foundations of learning: The principles of learning influence the choice of a teaching method capable of creating excitement and motivation, attracting students' attention to the lesson, taking into account individual differences, and determining the methods and tools of communication between the teacher and the student.

(Rifai, 2012, pp. 159-161)

Based on the previous criteria for choosing teaching strategies, and after the two researchers reviewed many educational studies and research related to active learning models and strategies, a suggested strategy was proposed according to active learning that suits the age stage, secondary school, financial and human capabilities, and the scientific material to achieve the desired goals, and its steps are:

First: preparation and integration of information

In this step, the students' previous information is determined that can be linked to the new knowledge, as:

- The school divides the class into groups of 6 or 8 students who are not homogeneous in academic level, naming each group with a name.
- The school randomly distributes worksheets with only one question for each group. The idea is for the groups to answer the questions posed and related to each other in a sequential manner to stimulate the students' previous information and link it to the focus of the new lesson.
- Members of each group work in pairs to discuss the answer, then the group as a whole discusses the answer for a period not exceeding 5 minutes.
- The school begins discussing the answers, starting from the first question with the group to which it was given, and ending with the last question with the last group. During the discussion of each question, the school displays an image of the question with the answer on the display screen in the form of a PowerPoint, thus forming an idea for the students about the previous information for the new lesson.

Second: Modeling and constructing meaning

The school provides examples in which it explains the most important new ideas that link the previous information to the subject of the lesson, enhancing this by displaying an educational video and real-life questions, and encouraging students to ask questions and inquiries, give examples, and model them in more than one image.

Third: Implementing ideas

At this stage, the school provides additional activities to support learning the lesson topic, including:

- Distribute worksheets to the groups containing the same question, and ask each group to answer it using the same mechanism in step (1).
- After the time is up, the answer is handed over to the school, which in turn chooses one of the answers and writes it on the board, then randomly chooses one student from each group to vote on the answer while providing an explanation and justification for the reason for his answer. The aim of this is to develop the decision-making process and responsibility for learning, as well as diagnose weak answers. And address them and ensure that students acquire the most important points of the lesson.

Fourth: Feedback

The school asks each group to write a summary of the most important issues covered in the lesson in the form of points or a conceptual outline. Then the school displays on the display screen a summary of the most important information and ideas covered in the lesson.

Mathematical representations

The process of learning and teaching school mathematics - in all its branches - is an interactive and cumulative process, relying on specific curricula, the basic requirement of which is that learning mathematics in it be based on understanding, to facilitate the acquisition of new knowledge (Al-Farisiya and Al-Hosnah, 2023, p. 2),Through this approach, the National Council of Teachers of Mathematics in the United States of America (NCTM, 2000) developed a global framework that contains standards for operations, and one of the most important of these standards is mathematical representations. The document indicates that mathematics programs from pre-kindergarten to the twelfth grade must enable all students to:

1. Building mathematical representations and using them to organize mathematical ideas and communicate them to others.
2. Choosing mathematical representations, applying them, and translating between them to solve problems.
3. Modeling and interpreting mathematical, natural, and social phenomena.

(NCTM,2000,p67)

(Abu Zeina, 2010, p. 103) believes that some forms of mathematical representations, such as graphs, pictograms, and verbal symbols, have long been part of school mathematics, However, it was taught as an end in itself. But representations should be treated as basic elements that support students' understanding of mathematical concepts and relationships, and the application and use of mathematics in real problem situations using modeling.

(Badawi, 2007, pp. 59-60) points out that students possess mathematical ideas and relationships using sensory materials, pictures, illustrations, graphs, tables, words, and symbols. This representation in its various forms helps them understand mathematical concepts and relationships, and most of those who deal with mathematical representations agree With what was stated in the document (NCTM, 2000), (Asli, 2001, p. 18) defined it as a mathematical embodiment of mathematical ideas and concepts to give the same information in more than one form, and (Al-Kubaisi and Abdullah, 2015, 133) saw it as internal abstractions of the mathematical idea, or a cognitive scheme that the learner may contribute to developing through experience.

The importance of mathematical representations

Mathematical representation is one of the most important main goals of the mathematics curriculum. Learning and teaching mathematics aims to represent situations mathematically and use the language of mathematics to express them. They are also effective tools that support mathematical ideas, as understanding concepts is enhanced when students are able to transfer understanding between different representations of the same idea (Fennel & Rowan, 2001, p. 289). The importance of mathematical representations lies in that they contribute to arranging and organizing mathematical ideas and processes, making them more sensory, and presenting mathematical concepts, relationships, and ideas in more than one representation, which is appropriate to the students' inclinations and mental abilities, and thus helps them build knowledge on their own and reach a deeper and better understanding (Al-Hanan, 2020, p. 238).

The importance of mathematical representations was stated by (Al-Kubaisi and Abdullah, 2015, p. 187), (Muhammad, 2015, p. 169), (Abu Hilal and Al-Astal, 2012, pp. 13-15) and (Abu Al-Ajen, 2011, p. 44) and can be summarized as follows:

1. It increases learners' ability to achieve and perform higher-order thinking processes.
2. It is considered a tool for mathematical communication, and a source of knowledge for learners.
3. It is used to model and explain natural, social and mathematical phenomena.
4. It makes it easier for learners to understand mathematical ideas through linking processes between different mathematical representations.
5. It employs the learner's different senses in the learning process and makes the mathematics lesson enjoyable.
6. It plays an important role in developing the learner's understanding of verbal mathematical concepts and problems.

The theoretical basis of mathematical representations

Mathematical representations are based on a strong theoretical foundation represented in the ideas of Bruner, Piaget, Deans, and Ausbel, as well as the theory of both sides of the brain.

Jean Piaget believes that children, through their interaction with their social and physical environments, advance their mental development through four successive stages: the sensorimotor stage, the pre-operational stage, the tangible operations stage, and the abstract formal operations stage. (Afana et al., 2010, p. p. 213-215), Bruner emphasized that mathematical representations occupy a fundamental position in cognitive development, and identified three methods that the individual needs to learn with representations: sensory representation, pictorial representation, and symbolic representation. Ozbel also divided learning into four types, arranged hierarchically from lowest to highest, as follows: (representational learning Learning concepts, learning issues, learning by discovery (Zaytoun, 2005, p. 592), Deans divided the representations into two parts: mathematical diversity (writing symbols) and appearance diversity (sensible objects), where the principle of appearance diversity indicates that learning the concept is deepened for the learner when it is processed through a variety of physical contexts, As for the principle of mathematical diversity, it refers to the generalization of a mathematical concept that is strengthened when viewed from different angles, such as 10% = 0.1 = 1/10 (Al Mahrezi and Al Ali, 2016, p. 48). One of the trends that dealt with mathematical representations is the theory of brain-based learning, as the left side of the brain represents the analytical aspect represented by words and symbols, while the right side of the brain is concerned with the structural aspects, such as shapes and drawings, and learning takes place through them in a complete and unified manner in which the two aspects overlap, It is necessary to focus on these two aspects in the teaching process by using many representations to stimulate learning among students (Abu Hilal and Al-Astal, 2012, p. 26).

The proposed model for mathematical representations.

There are many models of mathematical representations, perhaps the most important of which are:

Bruner's model: which consists of three types of representations: (practical representations, conceptual and imaginative (semi-sensory) representations, and symbolic representations) (Asteta, 2008, p. 248).

Lesh's model for mathematical understanding: Lesh's model can be considered an expansion of Bruner's theory, and this model consists of five components: (representation with pictures and shapes, representation with life situations, verbal representation, representation with written symbols, representation with models and objects) (Lesh, Post & Behr, 1987 ,p34).

Kaput model: Kaput has developed a model of mathematical representations that helps students with algebraic understanding. This model consists of three components: mathematical equations,, chart tables, and a graphs. (Kaput, 1989, p. 176).

Van de Walle et al's model for mathematical representations:

(Van de Walle et al., 2013, p45) presented a model for mathematical representations to improve mathematical understanding, which is an expanded and developed model of Lesch's model for mathematical representations and its components (life situations, pictures, data tables, graphs, verbal language, symbols, manuals).

The researchers decided to combine the two models (Kaput) and (Van de Walle) to suit higher stages, such as the secondary and university stages. This proposed model consists of eight components:

1. Verbal language: It is a means by which we express an idea, concept, relationship, equation, or mathematical generalization by speaking in a mathematical language that the student understands.

2. Written symbols: represent any means of expressing an idea, concept, relationship, equation, or mathematical generalization by writing it in the form of mathematical symbols.
3. Pictures and shapes: It is an educational tool that contains illustrative pictures or shapes that the student can see with his own eyes, or that the teacher draws or that the student draws, as it helps students learn a lot about mathematical ideas and concepts.
4. Artifacts (models and paradigm): expressing things with three-dimensional models to clarify the meaning.
5. Life situations: They represent situations and situations related to the student's real life and are linked to the concepts, relationships, equations, and mathematical generalizations given to the student.
6. Chart tables: Chart tables consisting of two columns
7. Graph: A type of visual representation, straight, curved, or inclined, of an algebraic rule, function, mathematical equation, or graphing tables in the Cartesian coordinate plane consisting of the X and Y axes.
8. Mathematical equations: They help the student express a mathematical idea or concept by linking two equal quantities, one or both of which contain variables X, Y and real coefficients, thus enabling him to represent them with tables and graphs.

The two researchers believe that what distinguishes this proposed model is flexibility and the ability to move between representational forms, not in a linear manner. Rather, it is possible to move from any form to another, or move internally within a single representational form, or move between two representations and another, which will reflect the impact of this on the students' way of thinking. And their understanding of mathematical concepts, ideas and situations well, as shown in the following chart:

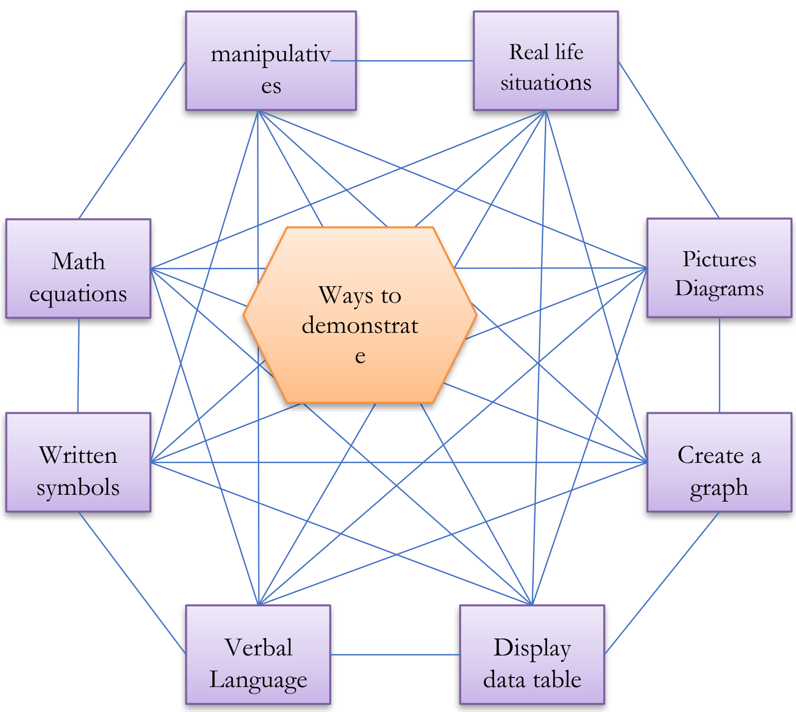

Figure 1: The diagram is the work of the two researchers

LITERATURE OF REVIEW

Studies dealing with active learning

1. Study (Badr, 2011)

The study was conducted in Saudi Arabia, and aimed to identify the effectiveness of teaching with a proposed strategy for active learning on developing higher-order thinking skills in engineering among low-achieving female students in the middle school. A quasi-experimental design with two experimental and control groups was used, and the sample consisted of (50) female students from the second year middle school. The research tool was to test higher-order thinking skills in engineering, and statistical means represented by arithmetic means, standard deviations, t-test, and the effect size equation were used. The results showed the effectiveness of the proposed strategy for active learning in higher-order thinking skills in engineering and for the benefit of the experimental group.

2. Study (Al-Deeb, 2020)

The study was conducted in Palestine, and aimed to identify the effectiveness of active learning in developing both achievement and mathematics teaching skills among a sample of female students in the College of Education and their attitudes towards it. A quasi-experimental design with two experimental and control groups was used, and the sample consisted of (110) female students, and (59) female students for the experimental group, and (51) female students for the control group, the tool was an achievement test with a note card to evaluate the students' performance, and a measure of the trend toward active learning strategies. Statistical means were used, represented by the Pearson correlation coefficient, (t-test), the Berman-Brown equation, and the ALPH-Cronbach equation. The results showed that the experimental group that was taught with active learning strategies was superior to the control group in achievement and attitude towards active learning strategies.

Studies dealing with mathematical representations

1. Study (Supandi el at, 2018)

The study was conducted in Indonesia, and the study aimed to identify the effect of the Think-Speak-Write model in improving mathematical representation and self-efficacy for eighth grade students. A quasi-experimental design was used, with experimental and control groups. The sample consisted of (64) students, with (32) for the experimental group And (32) for the control group. The research tool was the Mathematical Representations Test and the Self-Efficacy Test. Statistical means were used, represented by the Shapiroulk test, two-way analysis of variance, and the Scheffé test. The results showed that there was a statistically significant difference between the scores of the experimental and control groups in the Mathematical Representations Test and in favor of the Experimental group.

2. Study (Al-Ahwal, 2023)

The study was conducted in Egypt, and the study aimed to identify the effectiveness of the 9E learning cycle model using interactive learning media and its impact on improving cognitive learning outcomes and the ability of secondary school students to perform mathematical representations in solving verbal problems. A quasi-experimental design was used with a pre-post test, with two experimental and control groups. The sample consisted of (43) students, (23) for the experimental group and (20) for the control group. The research tool was a test of mathematical representation ability, and a test of cognitive learning outcomes. Statistical means represented by the SPSS program, the Cooper-Vacronbach equation, and the Pearson correlation coefficient were used. It showed The results showed a statistically significant difference in favor of the experimental group in the mathematical representations test.

Research methodology and procedures

First: Research methodology and design

The two researchers chose the experimental research method, including a quasi-experimental design for two experimental and control groups with a post-test, which is one of the real designs, as it represents active learning (the independent variable) and mathematical representations (the dependent variable).

Second: The research community was represented by all fifth scientific grade students in government secondary day schools affiliated with the General Directorate of Education in Baghdad/Al-Karkh II for the academic year 2023-2024.

Third: Research sample

The school (Zulfiqar Scientific secondarySchool for Boys) affiliated with the General Directorate of Education of Baghdad / Al-Karkh II was chosen intentionally,where the researcher works , The aforementioned school was visited, and it was found that the school includes five sections for the fifth scientific grade, distributed according to the letters of the alphabet (A, B, C, D, E). The fifth scientific section (D) was chosen by random drawing as an experimental group that will be taught according to the proposed strategy for active learning, as The number of its students reached (34), and the fifth scientific section (A) as a control group will be taught according to the usual method, as the number of its students reached (32) students.

Fourth: Control procedures

To ensure that the experiment proceeds correctly and clearly, some factors and variables that may affect the application of the experiment and the accuracy and validity of the results were controlled, before the actual teaching began as follows:

a. **Internal integrity of the experimental design**
   To verify the internal soundness of the experimental design, equivalences were conducted for the two research groups with some variables, namely (previous achievement in mathematics, previous mathematical knowledge, intelligence, and mathematical representations). In order to verify the equivalence of the students of the two research groups in the above variables, a one-way analysis of variance was used (ONE – Way – ANOV) the results showed that there were no statistically significant differences among the students of the two research groups.
b. **External validity of the experimental design (controlling extraneous variables).**
   To ensure the external validity of the experimental design, the two researchers controlled extraneous variables, including: the conditions of the experiment and the incidents accompanying it, the experimental extinction, the subject teacher, the academic content, the teaching plans, and the distribution of the lessons.

Fifth: Research requirements

It includes the following:

1. Propose steps for the proposed strategy according to active learning.
2. Determine the scientific material that will be studied during the experiment.
3. Analyzing the content of the mathematics book according to the components of mathematical knowledge.
4. Formulating behavioral objectives: (224) behavioral objectives were formulated based on Bloom's classification of the cognitive domain.
5. Preparing teaching plans: The researcher prepared (48) daily teaching plans for each group, with (96) plans for the two research groups.

Sixth: Preparing the research tool

The mathematical representations test was constructed according to the following steps:

1. Determine the goal of the test, which is to measure the mathematical representations of the two research groups.
2. Determine the mathematical representation model and its sub-indicators and present it to the arbitrators to reach its final form.
3. Analysis of the content of the first four chapters of the mathematics book/fifth scientific grade according to the indicators of the components of the proposed mathematical representations model.
4. Preparing the test map.
5. Formulating the test items, as (36) essay-type and objective-type items were drafted.
6. Preparing test and answer instructions.
7. Presenting the test items to the judges in mathematics and methods of teaching it.
8. The first exploratory application (information sample): It was applied to (23) students from the fifth scientific grade and not from the research sample to ensure the clarity of the instructions for the test items and to determine the time required for it. It was found that the instructions are clear and the test items are understandable to everyone, and the time was calculated. It took 100 minutes to answer.
9. The second exploratory application (statistical analysis sample): It was applied to (110) students from the fifth scientific grade and not from the research sample to ascertain the difficulty, ease, and discriminating power of the test items, as it became clear after applying the equation of difficulty and discriminating power that it was acceptable the psychometric properties of the test were also verified, which included all types of validity (face validity, content validity, and construct validity based on the Pearson correlation coefficient). The results were good. The stability of the test was also verified based on the Cronbach equation, and the stability of correction for the essay items.
10. The test in its final form: The test of mathematical representations in its final form consisted of (36) items, with (30) items of the objective, fixed-answer type and (6) items of the essay type. The test was applied to the two research groups on Monday, 1/15/ 2024, after students were informed a week before the test to prepare appropriately for it.

Seventh: Statistical means

The researchers used statistical methods represented by the Cooper equation, the Holsti equation, and using the statistical package Spss version 23, (one-way analysis of variance (One-Way-ANOV), Pearson correlation coefficient, Alpha-Cronbach equation, and Scheffé test) were extracted.

Presentation and interpretation of results

After applying the Mathematical Representations Test and correcting the students' answers, and to ensure consistency between the scores of the experimental and control groups in the Mathematical Representations Test, Levene's test was calculated, as it was found that the apparent significance level was (0.828), which is higher than the approved significance level (0.05), and this indicates that there is Homogeneity between the scores of the two research groups in mathematical representations, and the statistical package spss23 was also used to obtain data results for the experimental and control groups in the mathematical representations test, and the following table shows the results:

Table (1)

Groupes	Num ber	average	St. deviation	St.Error	t-test	tabular t-test	Sig f
Experimental group	34	77.382	11.160	1.913	5.814	2	64
Control group	32	61.781	10.606	1.874			

The arithmetic average of the experimental group's grades reached (77.382) degrees, while the arithmetic average of the control group's grades reached (61.781) grades. By comparing the arithmetic average of the experimental and control group's grades, we notice that the real performance of the control group students is lower than the real performance of the experimental group students. This means that the proposed strategy according to active learning has an effect on mathematical representations and is in favor of the second experimental group.

To verify this, a t-test was adopted for two independent samples, and the calculated t-test value was (5.814), which is greater than the tabular t-test value (2) at a significance level of (0.05) and with a degree of freedom (64).), and because the calculated one is higher than the tabulated one, then the null hypothesis is rejected and the alternative is accepted.

Interpretation of the results

The results showed that the experimental group outperformed the control group, and the reason may be:

1. Active learning helped students compete in presenting their ideas and gave them the opportunity to work in small, active groups to ask questions, evaluate, and discuss.
2. The proposed strategy according to active learning focused on the student and made him the focus of the educational process. Through practicing various educational activities without exception or fear of making mistakes, with the encouragement and support of the teacher.
3. The various stages of the proposed strategy according to active learning led to the consolidation of relationships between students and the school, and between the students themselves.

CONCLUSIONS

In light of the research results, the following can be concluded:

1. Active learning led to changing the learning environment, to an active, interactive, and cooperative environment between students and the school in an atmosphere of familiarity and joy.
2. Using the proposed strategy helped students develop and activate higher-order thinking processes, and gave them a meaningful approach by making them interpret new knowledge based on previous knowledge present in their cognitive structure.
3. Dividing the class into small groups led to creating a kind of constructive group work during practical application and increasing student participation.

RECOMMENDATIONS

1. The need for mathematics curriculum developers at different levels of education to pay attention to the importance of linking cognitive content with modern teaching principles and strategies that develop students' mathematical representations and thinking skills.

2. The need to hold workshops for teachers to familiarize them with designing activities for different mathematical representations and to encourage learners to design and build these activities.
3. Building various electronic educational platforms related to active learning, for the benefit of mathematics teachers, in which various active learning strategies are presented.
4. The necessity of activating model lessons based on active learning, and encouraging teachers to use active learning strategies because of their positive effects on learning.

PROPOSALS

In light of the results of the study, the researcher suggests the following:

1. Study the effect of the proposed strategy according to active learning on other dependent variables, such as visual thinking, mathematical communication skills, and at various other levels of study, including university students.
2. An analytical study of the availability of mathematical representations according to the proposed model in the content of the mathematics textbook for the different educational levels.

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