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RESEARCH ARTICLE

Primary Signal Processing and Algorithm for Selecting an Approximating Function in the Presence of a Noise Component

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ABSTRACT

The article proposes an algorithm and mathematical for choosing the degree of polynomial approximation of a noisy signal in order to isolate its useful component. The proposed algorithm implements a two-level analysis of a digital signal. At the first stage, the signal is divided into components. To determine the points of sharp changes in the signal shape, a multicriteria filtering method. Its use allows us to isolate a low component function, determine function outliers, and also detect areas of sharp inflection of the function. Next, a polynomial approximation of the input sequence is applied, limited by the sections specified at the first stage of the algorithm. The paper presents a rationale for choosing depending data on the approximation type of the input component and determining the points of sharp change. For example, use line of an image obtained in the IR range to demonstrate the operation of the algorithm. The algorithm is used to detect the boundaries of objects on image.

1. INTRODUCTION

Automation of processes is the most important stage for the formation of control and decision-making systems and devices [1, 2]. The modern world generates a lot of data, some of which are described by simple signals. Devices for their recording are built on the transformation of physical phenomena, which are converted into digital form for automation and decision-making. The process of obtaining and converting data is associated with introducing noise into the signal [3]. In addition, the signal itself may also contain a noise component associated with the superposition of other signals by their physical processes of the source itself. Elimination of interference is an important step, the result of which greatly affects the result. Both analog circuits and mathematical methods can be used as approaches to data preprocessing [4]. The use of a discrete approach allows for the expansion of the analysis capability, but introduces computational requirements for the calculation device [5]. Multicomponent and multilevel analysis allows for the extraction of a useful signal with the ability to detect features even in fairly large noise.

The paper proposes an approach to two-level analysis of the input digital signal with a noise component. Possible areas of application may include: sensors, acoustics, medicine, geology, space research, chemical industry, robotics, automation, etc.

The paper proposes an algorithm and mathematical justification for choosing the degree of polynomial approximation of a noisy signal in order to isolate its useful component. The proposed algorithm implements a two-level analysis of a digital signal. At the first stage, the signal is divided into components. To determine the points of sharp changes in the signal shape, a multi-criteria filtering method is used, based on the simultaneous minimization of the sum of the squares of the second order of the formed estimates of the series and the square of the deviation of the input implementation and estimate. The use of this approach with different influence coefficients allows us to isolate a low-component function, determine function outliers, and also detect areas of sharp inflection of the function. The average estimate obtained as a result of the first and second stages with different approximation functions is used as the output estimate. Using polynomial approximation also allows us to predict subsequent values, which can be used for forecasting. Next, a polynomial approximation of the input sequence is applied, limited by the sections specified at the first stage of the algorithm. The paper presents a rationale for choosing a function depending on the preliminary type of the input component and determining the points of sharp change. On a set of synthetic test data without a noise component and with its imposition, optimal parameters for the degree of polynomial approximation are determined. Tables are presented showing the dependence of the error change on the type of signal and the magnitude of the noise component. A line of an image obtained in the IR range is used as natural data to demonstrate the operation of the algorithm. The algorithm is used to detect the boundaries of objects in the image.

The paper has the following structure. The problem of choosing an approximation function in Section 2. The proposed algorithm is presented in Section 3. Section 4 describes some experimental results. Finally, conclusions are discussed in Section 5.

2. The problem of choosing an approximating function

Let the result of observing a random process be "pure signal + noise" $Y(t) = U(t) + V(t)$ its values were obtained $y_1, y_2, \dots, y_n$ at pre-set times $t_1 < t_2 < \cdots < t_n$. In this situation, noise components are usually considered $V(t_1), V(t_2), \dots, V(t_n)$ mutually independent and belonging to normal function $N(0, \sigma)$. In this paper, we require only the first of these conditions to be met, and replace the second with a much softer requirement of continuity of random variables $V(t_i), i = 1, 2, \dots, n$. Obviously, the components of the noisy signal $Y(t_1), Y(t_2), \dots, Y(t_n)$ "inherit" both properties postulated for noise components.

Let's find the estimate of the values $u_1, u_2, \dots, u_n$ clear signal $U(t)$ at the specified moments. For all integers $i \le 0$ and $i \ge n+1$ set $y_i = 0$. Having set any value of the natural parameter m, we estimate the vector$(u_1, u_2, \dots, u_n)$ linear combination of n-dimensional vectors $(1, 1, \dots, 1), \left(y_{1+j}, y_{2+j}, \dots, y_{n+j}\right), j = -m, \dots, -2, -1, 1, 2, \dots, m$. Let $c_0, c_{-m}, \dots, c_{-2}, c_{-1}, c_1, c_2, \dots, c_m$ – its coefficients; then for each $1 \le i \le n$ estimation x_i quantities u_i is expressed by the formula $x_i = c_0 + \sum_{\substack{j=-m \\ j \ne 0}}^{m} c_j y_{i+j}$. Quantities c_j we will determine by the method of least squares, i.e. from the condition of minimization of the function

$$f_m(\vec{c}) = \sum_{i=1}^{n} (x_i - y_i)^2 = \sum_{i=1}^{n} \left(c_0 + \sum_{\substack{j=-m \\ j \ne 0}}^{m} c_j y_{i+j} - y_i \right)^2 \tag{1}$$

on the set $R^{2m+1} = \{\vec{c} = (c_0, c_{-m}, \dots, c_{-2}, c_{-1}, c_1, c_2, \dots, c_m) \colon c_j \in R, j = 0, \pm 1, \pm 2, \dots, \pm m\}$.

Let us find a sufficient condition for the strong convexity of the function $f_m(\vec{c})$. For this purpose, we find half of its second differential $d^2 f_m = \sum_{j=-m}^{m} \sum_{k=-m}^{m} \frac{\partial^2 f_m}{\partial c_j \partial c_k} dc_j \, dc_k$; along the way we will obtain a system of equations for finding points of possible extremum $f_m(\vec{c})$. So, let's calculate all the first partial derivatives of this function and equate them to zero. We'll reduce the resulting equations by 2:

$$\frac{1}{2} \cdot \frac{\partial f_m}{\partial c_0} = \sum_{i=1}^{n} \left(c_0 + \sum_{\substack{j=-m \\ j \ne 0}}^{m} c_j y_{i+j} - y_i\right) = n c_0 + \sum_{\substack{j=-m \\ j \ne 0}}^{m} c_j \sum_{i=1}^{n} y_{i+j} - \sum_{i=1}^{n} y_i = 0;$$

$$\frac{1}{2} \cdot \frac{\partial f_m}{\partial c_k} = \sum_{i=1}^{n} \left(c_0 + \sum_{\substack{j=-m \\ j \ne 0}}^{m} c_j y_{i+j} - y_i\right) y_{i+k} = c_0 \sum_{i=1}^{n} y_{i+k} + \sum_{\substack{j=-m \\ j \ne 0}}^{m} c_j \cdot$$

$$\sum_{i=1}^{n} y_{i+j}y_{i+k} - \sum_{i=1}^{n} y_i y_{i+k} = 0, \qquad k = -m, \dots, -2, -1, 1, 2, \dots, m.$$

Having adopted the notations $\alpha_j = \sum_{i=1}^{n} y_{i+j}$, $\alpha_{jk} = \sum_{i=1}^{n} y_{i+j}y_{i+k}$, we arrive at a system of *2m*+1 linear equations with the same number of unknowns $c_0, c_{-m}, \dots, c_{-2}, c_{-1}, c_1, c_2, \dots, c_m$:

$$\begin{cases} nc_0 + \sum_{\substack{j=-m \\ j\neq 0}}^{m} \alpha_j c_j = \alpha_0 \\ \alpha_k c_0 + \sum_{\substack{j=-m \\ j\neq 0}}^{m} \alpha_{jk} c_j = \alpha_{0k}, k = \pm 1, \pm 2, \dots, \pm m. \end{cases} \qquad (2)$$

The solutions of this system are the extremum points of the function $f_m(\vec{c})$. Set

$$\frac{1}{2} \cdot \frac{\partial^2 f_m}{\partial {c_0}^2} = \frac{\partial}{\partial c_0}(nc_0 + \sum_{\substack{j=-m \\ j\neq 0}}^{m} \alpha_j c_j - \alpha_0) = n; \; \frac{1}{2} \cdot \frac{\partial^2 f_m}{\partial {c_k}^2} = \frac{\partial}{\partial c_k}(\alpha_k c_0 +$$

$$\sum_{\substack{j=-m \\ j\neq 0}}^{m} \alpha_{jk} c_j - \alpha_{0k}) = \alpha_{kk}, \; \frac{1}{2} \cdot \frac{\partial^2 f_m}{\partial c_0 \partial c_k} = \frac{1}{2} \cdot \frac{\partial^2 f_m}{\partial c_k \partial c_0} = \frac{\partial}{\partial c_k}(nc_0 + \sum_{\substack{j=-m \\ j\neq 0}}^{m} \alpha_j c_j - \alpha_0) = \alpha_k, k$$

$$= \pm 1, \pm 2, \dots, \pm m.$$

When,

$$\frac{1}{2} \cdot \frac{\partial^2 f_m}{\partial c_k \partial c_l} = \frac{\partial}{\partial c_l}\left(\alpha_k c_0 + \sum_{\substack{j=-m \\ j\neq 0}}^{m} \alpha_{jk} c_j - \alpha_{0k}\right) = \alpha_{kl}, 1 \le |k|, |l| \le m, k \ne l$$

(here it is taken into account that $\alpha_{lk} = \alpha_{kl}$). Thus,

$$\frac{1}{2} \cdot d^2 f_m = n d{c_0}^2 + \sum_{\substack{k=-m \\ k\neq 0}}^{m} \alpha_{kk} d{c_k}^2 + 2 \sum_{\substack{k=-m \\ k\neq 0}}^{m} \alpha_k dc_0 dc_k + 2 \sum_{\substack{k,l=-m \\ k,l\neq 0, k<l}}^{m} \alpha_{kl}\, dc_k dc_l$$

$$= \sum_{i=1}^{n} d{c_0}^2 + \sum_{i=1}^{n} \sum_{\substack{k=-m \\ k\neq 0}}^{m} y_{i+k}^2 d{c_k}^2 +$$

$$+2 \sum_{i=1}^{n} \sum_{\substack{k=-m \\ k\neq 0}}^{m} y_{i+k} dc_0 dc_k + 2 \sum_{i=1}^{n} \sum_{\substack{k,l=-m \\ k,l\neq 0;\, k<l}}^{m} y_{i+k} y_{i+l}\, dc_k dc_l$$

$$= \sum_{i=1}^{n} (d{c_0}^2 + \sum_{\substack{k=-m \\ k\neq 0}}^{m} (y_{i+k} dc_k)^2 + 2 \sum_{\substack{k=-m \\ k\neq 0}}^{m} dc_0 \cdot y_{i+k} dc_k +$$

$$+2 \sum_{\substack{k,l=-m \\ k,l\neq 0;\, k<l}}^{m} y_{i+k} dc_k \cdot y_{i+l}\, dc_l) = \sum_{i=1}^{n} \left(dc_0 + \sum_{\substack{k=-m \\ k\neq 0}}^{m} y_{i+k} dc_k\right)^2 \ge 0.$$

As we can see, the quadratic form $\frac{1}{2} \cdot d^2 f_m$ is reliably quasi-positive definite. In this case, the event $\{d^2 f_m = 0 \text{ only in } dc_k = 0, k = -m, \dots, -2, -1, 0, 1, 2, \dots, m\}$

$$s_0+\sum_{\substack{k=-m\\k\neq 0}}^{m} y_{i+k}s_k=0,\ i=1,2,\dots,n \qquad (3)$$

this is also equal to the event {the rank r of the matrix H of system (3) is equal to the number $2m+1$ of its unknowns} [6].

Everywhere below, condition (1) is considered to be satisfied. Let us show that then the equality $r=2m+1$ is satisfied with probability 1. Let us select in the matrix H a square submatrix $Z=(z_{ij})_{i=1}^{2m+1}{}_{j=1}^{2m+1}$, formed by rows with numbers from $m+1$ to $3m+1$ inclusive. The first column of the matrix Z consists of ones, and all its other elements have the form y_l. Let us associate Z with a matrix Q of the same size, the elements q_{ij} which we define by the following rule: if $j=1$, to $q_{ij}=0$, in case $j>1$ $q_{ij}=l$ provided that y_l there is a literal representation of the element z_{ij} matrices Z. Let us show that with probability 1 the determinants of all principal submatrices

$$Z_1=(z_{11}), Z_2=\begin{pmatrix} z_{11} & z_{12}\\ z_{21} & z_{22}\end{pmatrix}, Z_3=\begin{pmatrix} z_{11} & z_{12} & z_{13}\\ z_{21} & z_{22} & z_{23}\\ z_{31} & z_{32} & z_{33}\end{pmatrix},\dots, Z_{2m+1}=Z$$

(principal minors) of the matrix Z will be nonzero. It is easy to see that both in the rows and in the columns of the matrix Q the elements strictly increase (the exception is its first column, consisting of zeros). It follows that for $2\le k\le 2m+1$ the only smallest nonzero and the only largest element of the principal submatrix Q_k matrices$Q$ are, respectively, $q_{12}=1$ and q_{kk}. In particular, the only greatest element $q_{2m+1,2m+1}$ matrices$Q_{2m+1}=Q$ equal $4m+1$,which does not exceed n, by virtue of (1). Thus, the index of each element of the matrices Z of the form y_l satisfies the double inequality $1\le l\le n$. This means that all its elements that are not in the first column represent observed values of random variables $Y(t_1), Y(t_2),\dots,Y(t_n)$. By setting events $B_k=\{\det Z_k\neq 0\}$, we will prove the equalities by the method of mathematical induction $P(B_k)=1, k=1,2,\dots,2m+1$. The basis of induction is obvious: the inequality $\det Z_1=z_{11}=1\neq 0$ reliable, therefore, $P(B_1)=1$. Let us justify the induction step: assuming for some $1\le k\le 2m$ equal $P(B_k)=1$, to $P(B_{k+1})=1$. Let's first calculate the conditional probability $P(\overline{B}_{k+1}/B_k)$. Set $\det Z_{k+1}=\sum_{j=1}^{k+1} z_{k+1,j}W_j$ $-$decomposition of the determinant of matrices Z_{k+1} by the last line; here W_j- algebraic complement of an element $z_{k+1,j}$. In case of an event occurrence B_k, i.e. when $\det Z_k(=W_{k+1})\neq 0$ occurrence $\overline{B}_{k+1}=\{\det Z_{k+1}=0\}$ equal:

$$y_{q_{k+1,k+1}}(=z_{k+1,k+1})=-\frac{\sum_{j=1}^{k} z_{k+1,j}W_j}{\det Z_k}. \qquad (4)$$

Let us take into account the mutual independence of random variables $Y(t_i), 1\le i\le n$ and the inequalities we actually established above $q_{ij}<q_{k+1,k+1}$, when $1\le i\le k+1$; $2\le j\le k+1$; $|k+1-i|+|k+1-j|>0$. These circumstances allow us to assert that the right-hand side of (4), considered as a random variable, and the random variable $Y(t_{q_{k+1,k+1}})$ are independent (we emphasize that in this reasoning the possibility is excluded $\det Z_k=0$). We further note that by the time $(t_{q_{k+1,k+1}-1}+t_{q_{k+1,k+1}})/2$ the first value will already be played out, but the second one will not be played out yet. In this situation, the occurrence of the event $\overline{B}_{k+1}$ means that the second quantity takes on an already defined value (4). However, a continuous random variable $Y(t_{q_{k+1,k+1}})$ takes each specific value with zero probability. Therefore, $P(\overline{B}_{k+1}/B_k)=0$, what equality entails $P(B_{k+1}/B_k)=1$. By virtue of the probability multiplication theorem, $P(B_{k+1}B_k)=P(B_k)P(B_{k+1}/B_k)=1\cdot 1=1$. Event $B_{k+1}B_k$ favors the event B_{k+1}, so, $1=P(B_{k+1}B_k)\le P(B_{k+1})\le 1$. From this follows the equality we need $P(B_{k+1})=1$. So, $P(B_k)=1$ foe all $1\le k\le 2m+1$; in particular, $P(B_{2m+1})=1$. This means that the determinant of matrices $Z_{2m+1}=Z$, being one of the minors of order $2m+1$ matrices H, with probability 1 is different from zero. Since $\{\det Z\neq 0\}\subseteq\{r=2m+1\}$, for $P(r=2m+1)=1$.

Let's endow the linear space $R^{2m+1}=\{d\vec{c}=(dc_0, dc_{-m},\dots,dc_{-2}, dc_{-1}, dc_1, dc_2,\dots,dc_m): dc_j\in R, j=0,\pm1,\pm2,\dots,\pm m\}$ Euclidean norm $\|d\vec{c}\|=\sqrt{\sum_{k=-m}^{k=m} dc_k^2}$. In fact, we have proved that with probability 1 on the unit sphere S of this space the strict inequality holds $\frac{1}{2} \cdot d^2 f_m = \sum_{i=1}^{n} \left(dc_0 + \sum_{\substack{k=-m \\ k \neq 0}}^{m} y_{i+k} dc_k\right)^2 > 0$. Due to the continuity of the function $\frac{1}{2} \cdot d^2 f_m$ on the set S, it reaches its smallest at S to ε_1 and max ε_2 values, and $0 < \varepsilon_1 \leq \varepsilon_2$. Then the implication is true $\|d\vec{c}\| \neq 0 \Rightarrow \varepsilon_1 \leq \frac{1}{2} \cdot (d^2 f_m)\left(\frac{d\vec{c}}{\|d\vec{c}\|}\right) \leq \varepsilon_2$. Multiplying all parts of this double inequality by $2\|d\vec{c}\|^2$, we get a double inequality $2\varepsilon_1 \|d\vec{c}\|^2 \leq (d^2 f_m)(d\vec{c}) \leq 2\varepsilon_2 \|d\vec{c}\|^2$, meaning, by definition, a strong convexity of the function $f_m(\vec{c})$ [7]. It is known that for a strongly convex function with domain R^{2m+1}: 1) the concepts of points of possible extremum, points of local extremum and points of global extremum are equivalent [8]; 2) there is a single point of global (local, possible) extremum [7]. From this follows the final statement:

Let condition (1) be satisfied. Then with probability 1 the linear system (2) has a unique solution $\vec{c}^*$. The function $f_m(\vec{c})$ reaches its smallest value on R^{2m+1} only at the point $\vec{c}^*$. This point can be found either by solving system (2).

Let us present the results of applying the method presented in the article to three pure signals defined analytically. For each of them, two calculations were carried out – for $n = 50$ and $n = 100$ (data if presented in tables 1-6)/ In all cases, noise belonging to the law was superimposed on the pure signal $N(0, 1)$. The following notations are used below: $S_y = \sum_{k=1}^{n} (y_k - u_k)^2$, $S_x = \sum_{k=1}^{n} (x_k - u_k)^2$.

Tables 1. Examples with parameters: $\boldsymbol{u = \frac{(12-t)(7+t)(0.6+t)}{t^3+t^2+0.2t+20}}$**:** $\boldsymbol{n = 50;\ t_i = \frac{i}{2}, i = 1,2,3 \dots 50;\ S_y = 40,73}$

m	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
S_x	16.91	11.29	9.33	8.79	13.23	14.20	14.02	14.24	16.06	16.85	18.19	18.39	20.36	20.68	20.51

Tables 2. Examples with parameters: $\boldsymbol{n = 100;\ t_i = \frac{i}{4}, i = 1,2,3 \dots 100;\ S_y = 80.08}$

m	1	2	3	4	5	6	7	8	9	10	11	12	13
S_x	32.90	21.09	12.87	10.05	24.00	27.46	29.56	32.16	32.53	33.59	34.71	35.26	35.74

Tables 3. Examples with parameters: $\boldsymbol{u = e^{0.1t}\cos(t) - \sqrt{t}}$**:** $\boldsymbol{n = 50;\ t_i = \frac{i}{2}, i = 1,2,3 \dots 50;\ S_y = 43.03}$

m	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
S_x	47.73	38.02	33.33	26.06	18.34	13.69	15.03	15.21	17.85	18.64	22.02	23.28	23.23	24.80	25.75

Tables 4. Examples with parameters: $\boldsymbol{n = 100;\ t_i = \frac{i}{4}, i = 1,2,3 \dots 100;\ S_y = 112.43}$

m	1	2	3	4	5	6	7	8	9	10	11	12	13
S_x	58.73	42.96	40.34	42.47	46.94	48.24	44.32	42.14	41.77	42.25	49.32	51.53	51.20

Tables 5. Examples with parameters: $\boldsymbol{u = 128t^4 - 256t^3 + 160t^2 - 32t + 1}$: $\boldsymbol{n = 50;\ t_i = \frac{i}{50}, i = 1,2,3 \dots 50;\ S_y = 39.34}$

m	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
S_x	10.09	8.53	8.04	6.97	13.76	19.26	19.46	21.81	21.83	22.71	22.94	22.83	22.78	23.02	23.00

Tables 6. Examples with parameters: $\boldsymbol{n = 100;\ t_i = \frac{i}{100}, i = 1,2,3 \dots 100;\ S_y = 92.75}$

m		1	2	3	4	5	6	7	8	9	10	11	12	13
S_x		21.20	19.38	17.86	20.13	19.05	21.87	21.77	22.26	22.44	22.97	23.18	24.12	24.32

As we can see, all six examples show a significant decrease in the deviation from the pure signal as a result of approximation. The experiment did not reveal any recommendations for choosing the optimal value of the parameter m. We will only note that the experience of using the method and increasing the complexity of calculations suggests using m from 3 to 6. Most often, the best value falls on m up 3-4. Setting too large values of m does not improve the quality of approximation.

3. The Multi-Criteria adaptive processing algorithm

In my early works [9-11] I developed an approach to processing signals and two-dimensional matrices using a combined method of multicriterial evaluation. The method given in [9] is based on the use of a multicriterial objective function. The given solution allows for the simultaneous minimization of the sum of squares of finite differences of the first order of the generated estimates of neighboring rows and the sum of the squared differences of the deviation of the input implementation and the generated estimate. This function allows, by changing the parameter λ, μ, to regulate the weight of the criterion and set their priority depending on the problem being solved. The modified objective function for processing a two-dimensional signal (image) has the form:

$$\varphi\left(\bar{s}_{0,0}, \bar{s}_{0,1}, \ldots, \bar{s}_{0,n}, \bar{s}_{0,1}, \bar{s}_{1,1}, \bar{s}_{1,n}, \ldots, \bar{s}_{m,0}, \bar{s}_{m,1}, \ldots, \bar{s}_{m,n}\right) =$$
$$\sum_{i=0}^{m}\sum_{j=0}^{n}\left(\bar{s}_{i,j} - s_{i,j}\right)^2 + \lambda\sum_{i=1}^{m}\sum_{j=0}^{n}\left(\bar{s}_{i,j} - \bar{s}_{i-1,j}\right)^2 + \mu\sum_{i=0}^{m}\sum_{j=1}^{n}\left(\bar{s}_{i,j} - \bar{s}_{i,j-1}\right)^2 \qquad (5)$$

when: λ, μ - given positive multipliers, m – number of rows in the image, k – number of columns in the image, $s_{i,j}$ – input sequence, $\bar{s}_{i,j}$ – result of minimizing the function (smoothed value).

The transition to the one-dimensional case of expression (5) is carried out by zeroing the parameter j and the smoothing coefficient μ. Determining and improving the quality of data through primary signal processing based on the multicriteria method requires finding the following particular solutions:

$$\bar{s}_{i,j}^{k+1} = \bar{s}_{i,j}^{k} - 2\cdot\alpha\cdot\frac{1}{2}\cdot\frac{\partial\varphi(\bar{s}^k)}{\partial s_{i,j}} = \bar{s}_{i,j}^{k} - 2\cdot\alpha\left(c_{i,j}\cdot s_{i,j}^{k} - s_{i,j} - \lambda\left(s_{i-1,j}^{k} + s_{i+1,j}^{k}\right) - \mu\left(s_{i,j-1}^{k} + s_{i,j+1}^{k}\right)\right), \qquad (6)$$
$$i = 0,1,\ldots,m; \quad j = 0,1,\ldots,n; \quad k = 0,1,2,\ldots$$

when: $c_{i,j} = 1 + \lambda\left(sign(i) + sign(m-i)\right) + \mu\left(sign(j) + sign(n-j)\right)$, $\bar{s}_1 = \dfrac{\alpha\cdot\sum_{i=1}^{n}\gamma_{n-i+1}\cdot y_i}{\beta_n}$,

$\gamma_k = \sum_{j=0}^{k-1}\binom{k+j-1}{2j}\cdot\alpha^j$, $\beta_k = \sum_{j=1}^{k}\binom{k+j-1}{2j-1}\cdot\alpha^j$, adjustment weighting factor $\alpha = {}^{1}/_{1+4\lambda+4\mu}$.

The implementation of the algorithm for primary processing of two-dimensional signals and selection of an approximating function in the presence of a noise component will be carried out based on the block diagram shown in Figure 1a. The implementation of the object analysis block (Figure 1b) is based on the analysis of a string of values recorded in 8 directions relative to the studied image element. The formed string is analyzed in order to identify areas of local change in the function, with subsequent limitation of the selected range and its processing by the target function (1) in order to suppress interference.

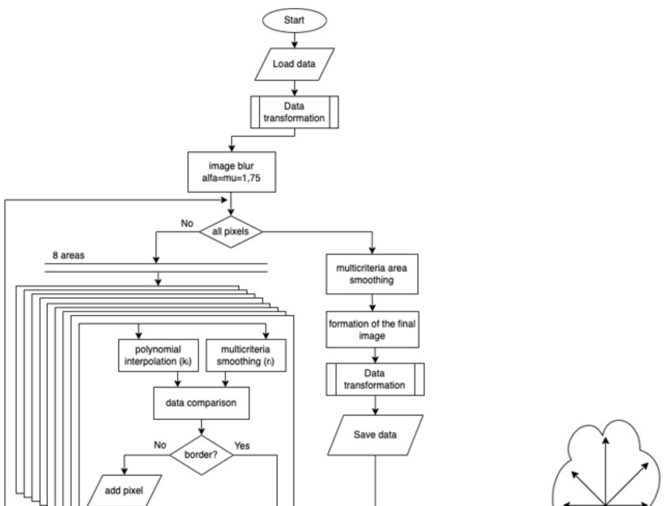

Figure 1. Block diagram of the algorithm for primary signal processing and selection of the approximating function in the presence of a noise component (Fig. a). Example of the formation of a data line and its visualization (b)

The algorithm proposed in Figure 1(a) allows processing signals with the possibility of adaptive selection of smoothing function parameters and formation of feature boundaries used to localize stationary analysis areas in images. The presented algorithm is implemented using the following main stages:

1. Loading data. Time series, signals or two-dimensional matrices can be used as analyzed data. In this paper, we use the approach to forming local stationary regions in images using the approach proposed in Block 2. As test data, we will use images recorded by a camera in the far infrared range (thermal signatures). The analyzed images are presented in grayscale with a color depth of 8 bits.

2. Blurring the image and selecting a random point on the image. At this stage, the entire image is processed using the multicriterial method using expressions (5) and (6) [10]. As a processing coefficient, we set the value λ, μ =1.75. Introducing this parameter as a smoothing one will allow for a slight blurring of the noise component and object boundaries. After global image processing, we select a random point on the image, while setting a limitation on its displacement from the boundaries at a distance of at least 5 pixels, which is a limitation for the polynomial approximation specified in Section 2.

3. Selecting the growth direction and conducting an analysis of the function divergence assessment. At this stage, we form a data line, which is obtained by increasing the value by the neighboring one in one of the directions shown in Figure 1b. The growth of the value will be stopped when the condition of reaching the object/image boundary is met or a sharp change in the function of the line is detected, which indicates that the object boundary has been reached.

4. Determination of the area of sharp change of the function. To fulfill this condition, the results recorded by the approach proposed in Section 2 with a polynomial of degrees 3 and 4 (parameter k_i) are compared, as well as the result of data processing by the objective function (5), with the processing parameters λ=4,4 (parameter r_i). Using a polynomial function of the function allows to smooth the signal significantly, an additional multicriteria component allows to reduce the noise component while maintaining the boundaries of sharp changes in the function. The detection criterion is the square of the degree of increment $\left|\frac{k_i}{r_i}\right| \leq \begin{cases} \left|\frac{k_{i-1}}{r_{i-1}}\right| + 0,05\left|\frac{k_{i-1}}{r_{i-1}}\right| \\ \left|\frac{k_{i-1}}{r_{i-1}}\right| - 0,05\left|\frac{k_{i-1}}{r_{i-1}}\right| \end{cases}$, where the indicator i is the final element of the increment of the row formed in step 2 of the current algorithm.

5. Localization of the area. At this stage, the area of interest is limited and excluded from further consideration. Then we return to stage 3 and randomly select a new point. The procedure is repeated until the image is completely covered or the condition for taking a new point of interest cannot be met.

6. Multicriterial processing of an image block. The generated local sections serve as a mask for processing the input image, which is processed in them using expressions (5) and (6) with the parameter λ, μ =8.7, which allows for a strong suppression of the noise component. The section with the region that did not fall into the localized region as a result of processing is replaced by those obtained at stage 2.

7. Saving the result. The image layers are combined and the final image is formed in the format of the input data.

The proposed approach allows forming parallel branches of the algorithm, at steps 3-5. Additionally, it is possible to save the mask of objects and use it for automated training of subsequent decision-making blocks.

4. EXPERIMENTAL RESULTS

To analyze the efficiency of the approaches proposed in the work, we will use the analysis of images recorded in the thermal range. The data are pre-converted into grayscale with a color depth of 8 bits. The frame resolution is 320x240 pixels. The analysis was carried out on a simple mobile platform with low computing performance on an IBM-compatible computer. An example of test processing of a normalized test line of an image is shown in Figure 2, the test data are shown in Figure 3, and the result of the processing is shown in Figure 4.

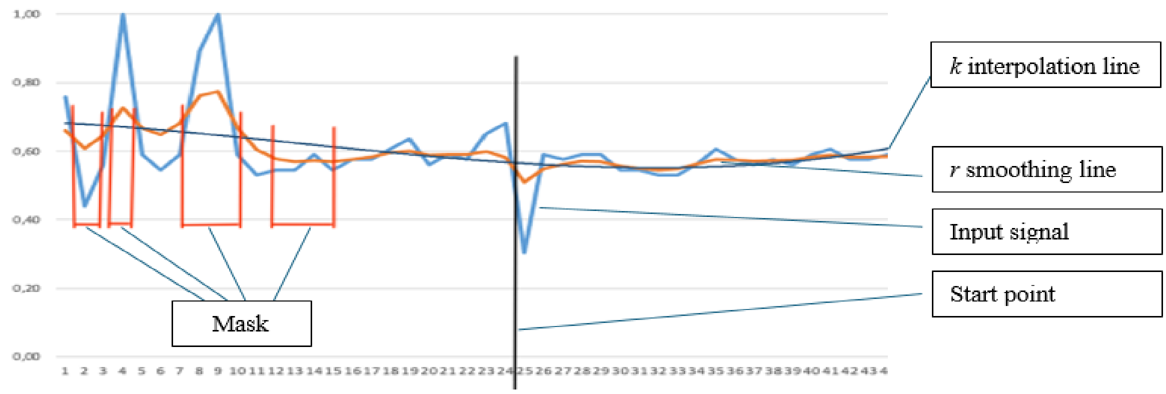

Figure 2. An example of processing an image line (blue line – input data; orange line – result of processing by a multicriterial method; black line – approximation by a 4th degree polynomial; red – detected areas/boundaries of objects)

Figure 3. An example test images 320x240 pixels in grayscale

Figure 4. Example of image processing

The example of image processing by a combined approach presented in Figure 4 demonstrated the applicability of the algorithms proposed in Chapters 2 and 3 for improving the quality of input two-dimensional signals. The example of processing a numerical sequence also demonstrated the possibility of detecting the boundaries of sharp changes in a function corresponding to the edges of objects recorded in images.

CONCLUSIONS

Modern systems often use streaming data received from various sensors. The process of receiving and converting data is associated with introducing noise into the signal. It should be noted that the data itself may have a non-stationary component, including the simultaneous registration of several streams. These data can be both interference and a useful signal, which at this stage is not needed for analysis. Thermal imaging is an example of such data. The recorded physical parameters are non-uniform, and the sensors are very sensitive to external influences. The presence of external factors in combination with the imperfection (non-ideality) of sensitive elements. The formation of new computationally simple approaches to reduce the effect of such interference is and will be an urgent task for a long period. The approach proposed in the work allows implementing parallel calculations with a combination of filtering algorithms. The use of local processing and global analysis allows us to reduce computational requirements and form a concept for building distributed big data systems. The next stage of development of the proposed implementations will be the direction of combined analysis of a color or multi-range two-dimensional signal. The areas of possible application are microelectronics, medicine, applied machine vision, etc.

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REFERENCES

1.Nezhmetdinov, R., Kotyrova, S., Nezhmetdinova, R., Kovalev, I. (2023) Automating log analysis for industrial equipment maintenance using elastic stack, E3S Web of Conferences, Volume 389, 01064 (2023).

2.Nezhmetdinov, R.A., Charuiskaya, M.A., Kovalev, I.A., Enterprise Development Planning and AI-Based Technological Forecasting, Russian Engineering Research, Volume 43, Issue 10, Pages 1284-1288 (2023).

3.Owotogbe, J. S., T. S. Ibiyemi, and B. A. Adu. "A comprehensive review on various types of noise in image processing." int. J. Sci. eng. res 10.11, P.388-393 (2019).

4.Al-Khaffaf, Hasan SM, Abdullah Z. Talib, and Rosalina Abdul Salam. "Removing salt-and-pepper noise from binary images of engineering drawings." 2008 19th International Conference on Pattern Recognition. IEEE, 2008.

5.Voronin, V., Zhdanova, M., Semenishchev, E., Zelenskii, A., Cen, Y., Agaian, S. "Action recognition for the robotics and manufacturing automation using 3-D binary micro-block difference." The International Journal of Advanced Manufacturing Technology, 117, pp. 2319-2330 (2021).

6.Ilyin, V.A. Linear Algebra: Textbook for Universities. – 4th ed. / V.A.Ilyin, E.G.Poznyak. – M. Science. Fizmatlit, 1999. – 296 p.

7.Ilyin, V.A. Fundamentals of Mathematical Analysis: In 2 Parts. Part I: Textbook for Universities. – 5th ed. / V.A.Ilyin, E.G.Poznyak. – M. Science. Fizmatlit, 1998. – 616 p.

8.Kashina, O.A. Optimization Methods. Part I / O.A.Kashina, A.I.Korablyov // Kazan Federal University – 2022.

9.Semenishchev, Evgeny, et al. "Image denoising using a combined criterion." Mobile Multimedia/Image Processing, Security, and Applications 2016. Vol. 9869. pp. 96-102 (2016).

10.Semenishchev, E., Shraifel, I., Marchuk, V., Svirin, I., Makov, S. “A multi-criteria method for noise reduction.” In 2016 IEEE East-West Design & Test Symposium (EWDTS), pp. 1-6, IEEE (2016).

11.Semenishchev, E. “Algorithm combination of deblurring and denoising on video frames using the method search of local features on image.” In MATEC Web of Conferences, Vol. 132, p. 05018 (2017).