



## Pakistan Journal of Life and Social Sciences

www.pjlss.edu.pk

### RESEARCH ARTICLE

## Forecasting the Infant Mortality Using Time Series Models

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### ARTICLE INFO

Received: Dec 20, 2020

Accepted: June 20, 2021

### Keywords

Infant Mortality  
Bottom-up  
Top-down  
Optimal Combination  
ARIMA  
Prediction

### ABSTRACT

Infant Mortality has always remained the most important area of the global population; Specifically, the reduction in infant mortality. The secondary data about Total Infant Mortality (TIM) in Punjab, Sindh, Balochistan, and Khyber-Pakhtunkhwa during 1968-2017 have been analyzed and interpreted in this study. The data during 1968-2007 are treated as a training set whereas during 2008-17 as a test set. The means of female TIM of Punjab, Sind, KPK and Balochistan are 79678, 28448, 17531, and 8758.6 respectively while the means of male TIM of the same order are 98264, 33098, 23258, and 8832.5 respectively. The Standard Deviation (SD) of Punjab female TIM is maximum i.e., 12543 and KPK female TIM is 3073.6 while the SD of Punjab male TIM is Maximum i.e., 20396 and Balochistan male TIM is 2912.2 only. It indicates the strong genetic makeup and better immune system of females than males. It might be because of the population density also. TIM of Pakistan was predicted using Bottom-Up, Top-Down and Optimal Combination techniques taking ARIMA as the base. The goodness of fit was also tested using MPE, MAPE and RMSE. The smaller values of RMSE for female Punjab, male Sindh, male-female KPK and male-female Balochistan are 3905.38, 6377.55, 3453.44, 3088.42, 2026.49 and 2224.30 respectively illustrating that Optimal Combination technique fitted better than the Bottom-Up and Top-Down techniques. For comparison purposes, TIM was also forecasted during 2018-27 using the said three techniques. However, Optimal Combination forecasting was found to be more consistent than the Bottom-Up and Top-Down techniques and may be preferred.

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### INTRODUCTION

Infant mortality rate (IMR) may be referred to the probability of occurrence of a baby's death on any day before or on his/her first birthday. It is an enduring problem related to the public health. IMR is the indispensable measure to define the demographic and socio-economic status of a country and the quality of life of its people [UNDP, 2007]. As per the Sustainable Development Goal [SDG], the reduction in IMR is a vital aim to achieving universal health (United Nations, 2018). According to the Central Investigation Agency World Factbook (2017-2018), Pakistan stands at the 20<sup>th</sup> position based on IMR. UNICEF (2018) stated that Pakistan was the country with "the worst new-born mortality rate" and with the chance of 1 death out of 22

infants. There is a 50 times more chance of death of a new-born baby in Pakistan within one month of birth as compared to a baby born in Singapore, Iceland or Japan. The executive director, of UNICEF, stated that the health of older children improved with time but the death of world poorest new-born babies within the first month could not be controlled. Islam and Hyder (2016) found that measures wrought for reduction in infants' death. However, the deaths of infants and their health conditions still stood significant topics in "South-Asian" countries. IMR was 78 in 2006-07 which decreased to 74 and 62 in 2012-13 and 2017-18 respectively. Despite modest improvement in Pakistan, the IMR is still considerably more as compared to other neighbouring countries in the region. It was reported that the Neonatal Mortality Rate (NMR) in rural areas

was 62 live births per 1,000 and 47 live births per 1,000 in urban areas of Pakistan whereas the death and birth rate of children under five years were recorded 57% and 43% respectively except those births and deaths that occurred at homes and unregistered (NIPS, 2017-2018). UNICEF also launched a campaign for literacy, awareness and health of infants (Khan et al., 2012). The neighbouring countries in the region, including Bangladesh and Sri Lanka, have improved the health standards and facilities. On the other hand, due to the lack of good Governance, transparency, justice and a politically driven budgets since long that led to poor health conditions and more deaths of the infants (Ghaffar et al., 2000; Hakro, 2007). The World Health Organization (WHO) suggests and recommends that all the member countries should allocate at least 5% of their GDP to their health sector to achieve their targets of infant mortality decline (Savedoff, 2007). The Low-income women group and the poor health condition of the children also led to the infant mortality (Agha, 2000). Parents education also plays a significant role in IMR, the less educated they are, the higher will be the risk of infant death (Pradhan and Arokiasamy, 2010; Chalasani, 2012). Improved health and nutritional conditions along with lesser rural population may be supportive to reduce the infant mortality (Bhutta et al., 2013). Malnutrition was considered one of the major causes of mortality i.e., 194 (24.6%) whereas the others Acute Respiratory Infections as 188 (23.9%) and Diarrheal Disease 161 (20.4%) (Zafar, 2011). On the other hand, lack of awareness, lack of transportation facilities, illiteracy, poverty, absence of antenatal care services and home deliveries by unskilled lady workers are also the main causes of infant mortality (Khan et al., 2020). These socio-economic and demographic characteristics may play a significant role in extensive information and policymaking, as well as improved performance & accuracy (Gonzalez and Gilleskie, 2017). Rashid and Chand (2016) brought forward the spatial analysis of infant mortality rate in Punjab by determining the related nature of infant mortality and the high-risk cluster districts of Punjab, using MICS 2007-08. The results exhibited that the conditional autoregressive time series model seemed to be better than the others. Hyndman and Koehler (2006) conducted fast computation to reconcile hierarchical Australian Labour Force censuses data, the data spread over the 3<sup>rd</sup> quarter of 1986 to the 2<sup>nd</sup> quarter of 2013. It was concluded that a fast algorithm for grouped time series and sparse matrix solutions may be preferred for all cases of data. Shang (2017) studied the Infant Mortality Rates in Australia nationwide and sub-national level wise. Grouped time series methods i.e., Bottom-Up and Optimal Combination were fitted. The Pointwise and averaged prediction interval were also

constructed. Mircetic et al. (2017) proposed modification in a Top-Down approach for hierarchical forecasting. The Top-Down approach was projected on the ratio of the bottom and top-level series. For the beverage supply chain, a new method along with simulation performed better. Athanasopoulos et al. (2017) introduced temporal hierarchy for time series phenomena. This method of time series is non-overlapping temporal aggregation. The researchers concluded that hierarchical time series worked better than conventional forecasting. Shang and Haberman (2017) aggregated point forecasts for different disaggregated factors using bootstrapping. Point and interval forecasting were also designed up to 15 steps ahead using independent and grouped functional time series. However, the grouped methodology shaped the estimates closer. Gao and Shang (2017) used Switzerland and the Czech Republic data for the year 1950 to 2014 and applied the principal component analysis and vector error correction statistical techniques considering the property of randomness with the non-parametric smoothing. For the interval forecast, non-parametric bootstrap method was also used. Ijaz et al. (2018) discussed the infant mortality in Peshawar region using the cross-sectional primary data over the period January 2017 and April 2017. The study revealed the high mortality i.e., 98 out of 1000 live births and 54% of the total deaths were sudden or due to gastroenteritis or pneumonia. In addition, 63 out of 1000 Infants died, who were born to young mothers who fell in the age bracket of 15 to 20 years. Arokiasamy and Anbuose (2018) deliberated the psychological discomfort caused by infant mortality rates and behavioural studies using structural equations modelling. The researcher also discussed a new approach for the improvement of health services. Garcia et al. (2019) analysed the risk factors for the neonatal and infants using the hierarchical logistic regression and discussed the association among demographic, socio-economic, behavioural and health services characteristics with that of the death. Shang (2016) discussed the available age-specific mortality data of the United Kingdom and Australia for the periods 1922-2009 and 1950-2003 respectively using the multi-level functional data methods and concluded that multi-level functional data method was more precise than the Bayesian method. Keeping in view the global concern about the gravity of infant mortality in general and specifically in Pakistan, this study is designed to estimate and predict infant mortality for the upcoming years using a Parsimonious Hierarchical Time Series Model as well as the goodness of fit of the predicted model. The data on infant mortality in counts have been taken from the most authentic sources in Pakistan i.e., National Institute of Population Studies

[NIPS] reports of the year(s) 2012-2013, 2017-2018, Pakistan Demographic Survey [PDS] from 1980 to 2007 and population growth survey 1968-1970, published by Pakistan Bureau of Statistics [PBS]. The Infant mortality data about four provinces of Pakistan have been extracted for different periods and levels from the reports and utilized in this study. Furthermore. The executed methods for the study under hierarchical time series were labelled as an Optimal Combination, Bottom-Up and Top-Down. The main objective of the study was to forecast the Infant Mortality using the parsimonious hierarchical time series model.

## MATERIALS AND METHODS

### Data description

The most authentic and reliable secondary data about infant mortality total in counts gender as well as province wise available in Pakistan were utilized in this study i.e., Pakistan Bureau of Statistics [PBS] and Demographic and Health Survey [DHS] published by the National Institute of Population Studies [NIPS]. The total infant mortality male and female data were spread over the period 1968-2017. Out of which, the data for the period 1968-2007 were used as a training set and the remaining data for the period 2008-2017 as a test set. The nature of data used in this study is time-series.

### Statistical analysis

#### Forecasting hierarchical or grouped time series

Time series can often be naturally disaggregated by various attributes of interest.

#### Hierarchical time series

Hierarchical time series, based on disaggregated time series, usually require the forecasts to add up in the same way as the data. The hierarchical structure of this model can be written as

$$\hat{y}_t = S\hat{b}_t \quad 2$$

#### Grouped time series

Grouped time series involves more general aggregation structures. With grouped time series, the structure does not naturally disaggregate in a unique hierarchical manner, and often the disaggregating factors are both nested and crossed. The grouped structure may be presented as  $y_t = Sb_t$

#### Bottom-Up approach

Bottom-Up approach is a method for generating coherent forecasts, first, it involves generating forecasts for each series at bottom-level, and then summing to produce forecasts for all series in structure. Symbolically, it can be written as

$$\hat{y}_h = S\hat{b}_h \quad 3$$

#### Top-Down approach

Top-Down approaches work only with strictly hierarchical aggregation structures, not with the grouped structures. They involve first generating forecasts for the total series and then disaggregating these down the hierarchy. Top-Down approaches can be symbolized as

$$\hat{y}_h = Sp\hat{y}_t \quad 4$$

#### Optimal Combination

Optimal Combination forecast reconciliation will occur, if the researcher can find the G matrix which minimises the forecast error of the set of coherent forecasts using

$$\hat{y}_h = SG\hat{y}_h \quad 5$$

Wickramasuriya et al. (2019) gave the variance-covariance matrix of the h-step-ahead coherent forecast as

$$\hat{y}_h = S(S'W_h^{-1}S)^{-1}S'W_h^{-1}\hat{y}_h \quad 6$$

(Hyndman and Athanasopoulos, 2018, chap 10)

#### Model Efficiency Criteria

##### AIC: Akaike Information Criterion

Mathematically, the Akaike information criteria for model selection may be written as  $AIC = \ln \sigma_k^2 + \frac{n+2k}{n}$ , 7

Where  $\sigma_k^2 = \frac{RSS_k}{n}$ , k and n are the number of parameters and number of observations respectively (Akaike 1973, p. 267).

##### BIC: Bayesian Information Criterion

BIC is also a criterion for model selection based on the likelihood function,

$$BIC = -2*LL + \log_e(N)*K \quad 8$$

Where  $\log_e$  is the natural logarithm, LL is the log-likelihood of the model, N is no of examples in the training data set and k is no of parameters in the model (Schwarz, 1978, p: 461).

##### RMSE: Root Mean Square Error

Adhikari and Agrawal (2013, pp: 43-45) described the following accuracy measures i.e. RMSE is a frequently used accuracy measure to find out the differences between observe and predicted values. Symbolically, it may be written as

$$RMSE = \sqrt{\frac{\sum_{t=1}^T (Y_t - \hat{Y}_t)^2}{T}} \quad 9$$

Where  $Y_t$  and  $\hat{Y}_t$  are the observed and predicted values respectively

##### MAE: Mean Absolute Error

It measures the correctness of the prediction. The positive and negative error effects remain the same because of absolute. Mathematically, it can be written as

$$MAE = \frac{1}{T} \sum_{t=1}^T |Y_t - \hat{Y}_t| = \frac{1}{T} |\varepsilon_t| \quad 10$$

Where  $Y_t$  and  $\hat{Y}_t$  are the observed and predicted values respectively.

**ME: Mean Error**

It is an informal term that bring up to the average of all the errors in a set. Mathematically, it can be written as

$$ME = \frac{1}{T} \sum_{t=1}^T (Y_t - \hat{Y}_t) = \frac{1}{T} \varepsilon_t \quad 11$$

Where  $Y_t$  and  $\hat{Y}_t$  are the observed and predicted values respectively.

**MAPE: Mean Absolute Percentage Error**

It is used to measure the model efficiency and states the correctness as percentage. Symbolically, it may be written as

$$MAPE = \frac{1}{T} \sum_{t=1}^T \left| \frac{Y_t - \hat{Y}_t}{Y_t} \right| \times 100 = \frac{1}{T} \sum_{t=1}^T \left| \frac{\varepsilon_t}{Y_t} \right| \times 100 \quad 12$$

Where  $Y_t$  and  $\hat{Y}_t$  are the observed and predicted values respectively.

**MPE: Mean Percentage Error**

It describes the percentage of average error. The smaller MPE indicates the trustworthy forecast. It may be inscribed as

$$MPE = \frac{1}{T} \sum_{t=1}^T \frac{Y_t - \hat{Y}_t}{Y_t} \times 100 = \frac{1}{T} \sum_{t=1}^T \frac{\varepsilon_t}{Y_t} \times 100 \quad 13$$

Where  $Y_t$  and  $\hat{Y}_t$  are the observed and predicted values respectively.

**MASE: Mean Absolute Scaled Error**

Hyndman and Koehler (2006, p: 680) illustrated that it is generally a suitable measure for forecast correctness. It scaled the errors i.e., positive and negative. Symbolically, it can be written as:

$$MASE = \frac{\frac{1}{T} \sum_{t=1}^T |Y_t - \hat{Y}_t|}{\frac{1}{T-1} \sum_{t=2}^T |Y_t - Y_{t-1}|} = \frac{\frac{1}{T} \sum_{t=1}^T |\varepsilon_t|}{\frac{1}{T-1} \sum_{t=2}^T |Y_t - Y_{t-1}|} \quad 14$$

Where  $Y_t$  and  $\hat{Y}_t$  are observed and predicted values respectively.

**RESULTS AND DISCUSSION**

This section will provide the complete analysis of this research along with the coherence that of the other studies in this area. Table 1 reveals the summary statistics of infant mortality totals of four provinces of Pakistan i.e., Punjab, Sindh, KPK and Balochistan during 1968-2017. It indicates that the maximum value for female total infant mortality in provinces Punjab, Sindh, KPK and Balochistan are 111000, 47476, 37222 and, 20500 and the minimum female TIM are 55400, 9414, 13921, 1740 along with standard deviations 12543, 6938.6, 3073.6 and 4459 respectively whereas maximum male total infant mortality in some provinces are 135008, 52800, 37200 and, 15400 and minimum TIM is 15500, 14377, 14400, 1490 along with a standard deviation are 20396, 9763.3, 6226.7 and 2919.2 respectively. The maximum standard deviation of female TIM is 12543 of province Punjab and the minimum standard deviation is 3073.6 in the province of KPK. On the other hand, Standard deviation of male TIM is 20396 which is the maximum of province

Punjab whereas Balochistan has a minimum standard deviation i.e., 2912.2. In general, the male TIM is greater than female in four provinces. As far as variation, province Punjab has greater variation as compared to the other provinces. The means of female total infant mortality of Punjab, Sind, KPK and Balochistan are 79678, 28448, 17531 and 8758.6 respectively. Similarly, the means of male TIM of same sequence are 98264, 33098, 23258 and 8832.5, respectively. It indicates that the female infant mortality in all provinces is less than the male TIM. It might be because of the female have better genetic make-up and Immune system by nature as compared to male genetic make-up. The skewness of the females with same sequence as 0.221, -0.299, 0.294, -0.158 and for males as 0.11, 0.189, 0.472, -0.320 respectively. Similarly, the smaller values of kurtosis of female TIM for Punjab, Sindh, KPK and Balochistan are -0.593, -1.158, 1.288 and -0.237 and for male with same sequence are -1.063, -0.275, -0.605 and 0.192, respectively. The small values of skewness and kurtosis both in male and female among four provinces indicate a slight departure from normality of the data and in coherence with the study (Westfall, 2014).

**Hierarchical Time Series**

In hierarchical time series analysis, the data have been distributed into four levels. The first level presents total infant mortality data, second level discriminates the data by male and female, third level elaborates the infant's data by province wise whereas the fourth level dispenses the infant data w.r.t to gender and province wise simultaneously.

Table 3 shows the upshots of Bottom-Up, Top-Down and Optimal Combination for male & female infants of each province separately taking ARIMA as base and Goodness of fit accuracy measures i.e., MPE, MAPE and RMSE. The smaller values of RMSE for female Punjab, male Sindh, male female KPK and male female Balochistan are 3905.38, 6377.55, 3453.44, 3088.42, 2026.49 and 2224.30 respectively illustrated that Optimal Combination technique fitted better than the Bottom-Up and Top-Down techniques. Although, the slight departure has been inspected in the male Punjab and female Sindh and suggested the Top-Down technique. The other two accuracy measures MPE and MAPE also depicted the similar conclusions with some variation towards Bottom-Up and Top-Down techniques. However, all these accuracy measures are the robust estimates and slight departure may be tolerable. Consequently, the preference may be given to the Optimal Combination technique due to its consistency. Optimum Combination technique is closer to Top-Down technique than the Bottom-Up. The smaller estimates of the accuracy measures are presented in bold face Table 3. Table 4 discovered the Total Infant Mortality forecasts comparisons during the

**Table 1: Summary Statistics of Total Infant Mortality of Pakistan Province and Gender wise during 1968-2017**

| Summary Statistics | Punjab |        | Sindh  |        | KPK    |        | Balochistan |        |
|--------------------|--------|--------|--------|--------|--------|--------|-------------|--------|
|                    | Female | Male   | Female | Male   | Female | Male   | Female      | Male   |
| Mean               | 79678  | 98264  | 28448  | 33098  | 17531  | 23258  | 8758.6      | 8832.5 |
| Median             | 79820  | 95038  | 29012  | 34152  | 17128  | 21974  | 9863.5      | 9209   |
| Maximum            | 111000 | 135008 | 47476  | 52800  | 37222  | 37200  | 20500       | 15400  |
| Minimum            | 55400  | 15500  | 9414   | 14377  | 13921  | 14400  | 1740        | 1490   |
| SD                 | 12543  | 20396  | 6938.6 | 9763.3 | 3073.6 | 6226.7 | 4459        | 2919.2 |
| Skewness           | 0.221  | 0.11   | -0.299 | 0.189  | 0.294  | 0.472  | -0.158      | -0.320 |
| Kurtosis           | -0.593 | -1.063 | -1.158 | -0.275 | 1.288  | -0.605 | -0.237      | 0.192  |

**Table 2a: Predicted Total Infant Mortality using Bottom-up Technique and Accuracy Measures**

| Years | $Y_t$  | Predicted TIM |        | Residuals | RMSE    | MAE     | MAPE |
|-------|--------|---------------|--------|-----------|---------|---------|------|
|       |        | Exponential   | ARIMA  |           |         |         |      |
| 2008  | 321300 | 213116.4      | 300400 | 20899.6   | 20899.6 | 20899.6 | 0.0  |
| 2009  | 298400 | 213280.5      | 301634 | -3234.2   | 14954.1 | 12066.9 | 3.7  |
| 2010  | 275400 | 213844.7      | 298979 | -23579.1  | 18286.8 | 15904.3 | 5.3  |
| 2011  | 272800 | 214208.8      | 297340 | -24540.0  | 20033.9 | 18063.2 | 6.2  |
| 2012  | 243100 | 214573.0      | 241550 | 1549.6    | 17932.3 | 14760.5 | 5.1  |
| 2013  | 239300 | 214937.1      | 214051 | 25249.3   | 19344.9 | 16508.6 | 6.0  |
| 2014  | 237500 | 215301.3      | 214040 | 23459.6   | 19984.7 | 17501.6 | 6.6  |
| 2015  | 233900 | 215665.4      | 214018 | 19881.6   | 19971.8 | 17799.1 | 6.8  |
| 2016  | 225500 | 216029.6      | 214058 | 11442.4   | 19212.0 | 17092.8 | 6.6  |
| 2017  | 209900 | 216393.7      | 213971 | -4071.1   | 18271.5 | 15790.6 | 6.1  |

**Table 2b: Predicted Total Infant Mortality using Top-Down Technique and Accuracy Measure**

| Year | $Y_t$  | Predicted TIM |        | Residuals | RMSE    | MAE     | MAPE |
|------|--------|---------------|--------|-----------|---------|---------|------|
|      |        | Exponential   | ARIMA  |           |         |         |      |
| 2008 | 321300 | 209964.4      | 292019 | 29281.3   | 29281.3 | 29281.3 | 0.0  |
| 2009 | 298400 | 210904.4      | 281278 | 17122.5   | 23985.1 | 23201.9 | 7.4  |
| 2010 | 275400 | 210966.4      | 280995 | -5594.9   | 19848.4 | 17332.9 | 5.6  |
| 2011 | 272800 | 210988.4      | 280887 | -8087.1   | 17658.4 | 15021.4 | 4.9  |
| 2012 | 243100 | 211623.4      | 270846 | -27746.1  | 20085.4 | 17566.3 | 6.2  |
| 2013 | 239300 | 212640.4      | 267830 | -28530.4  | 21722.1 | 19393.7 | 7.1  |
| 2014 | 237500 | 209964.4      | 260824 | -23324.4  | 21958.1 | 19955.2 | 7.5  |
| 2015 | 233900 | 210904.4      | 254022 | -20122.2  | 21737.1 | 19976.1 | 7.7  |
| 2016 | 225500 | 210966.4      | 240821 | -15321.3  | 21120.7 | 19458.9 | 7.6  |
| 2017 | 209900 | 210988.4      | 236821 | -26921.0  | 21770.4 | 20205.1 | 8.1  |

**Table 2c: Predicted Total Infant Mortality using Optimal Combination Technique and Accuracy Measure**

| Years | $Y_t$  | Predicted TIM |        | Residuals | RMSE    | MAE     | MAPE |
|-------|--------|---------------|--------|-----------|---------|---------|------|
|       |        | Exponential   | ARIMA  |           |         |         |      |
| 2008  | 321300 | 210791.4      | 292534 | 28766.0   | 28766.0 | 28766.0 | 0.0  |
| 2009  | 298400 | 210864.6      | 291794 | 6606.2    | 20870.1 | 17686.1 | 5.5  |
| 2010  | 275400 | 210937.9      | 281018 | -5618.4   | 17346.3 | 13663.5 | 4.4  |
| 2011  | 272800 | 211011.1      | 279095 | -6294.5   | 15348.5 | 11821.2 | 3.8  |
| 2012  | 243100 | 211084.3      | 265569 | -22469.4  | 17012.8 | 13950.9 | 4.9  |
| 2013  | 239300 | 211157.6      | 264836 | -25536.0  | 18705.0 | 15881.7 | 5.9  |
| 2014  | 237500 | 211230.8      | 250261 | -12761.4  | 17976.6 | 15435.9 | 5.8  |
| 2015  | 233900 | 211304.0      | 245556 | -11656.0  | 17313.2 | 14963.4 | 5.7  |
| 2016  | 225500 | 211377.3      | 230754 | -5254.1   | 16416.7 | 13884.6 | 5.3  |
| 2017  | 209900 | 211450.5      | 226732 | -16832.0  | 16458.6 | 14179.3 | 5.6  |

Table 2a-c present the observed total infant mortality year wise, predicted TIM using exponential and ARIMA as base, residuals and accuracy measures i.e., RMSE, MAE and MAPE for the years 2008-17 using Bottom-Up, Top-Down & Optimal Combination techniques respectively. According to the Bottom-Up technique, the pattern executed almost stable after 2013 whereas Top-Down & Optimal Combination findings indicate a judicious decline in TIM. The predictions by ARIMA as base illustrates a declining trend of total infant Mortality and come to an agreement with the observed TIM based on the RMSE. On the other hand, the predictions taking exponential as base do not coherence with the observed data. Shang (2016) used the multilevel functional data methods using the age and sex specific data of UK and preferred the Optimal Combination than the other two methods. Hyndman et al. (2011) also preferred the Optimal Combination in many applications, specifically, used the travel and geographical region time series data. The deductions of this study are consistent with the studies (Shang, 2017; Pervez et al., 1991; Hyndman et al., 2011; Shang and Smith, 2013). Khan et al. (2012) findings and suggestions in this regard are also very significant.

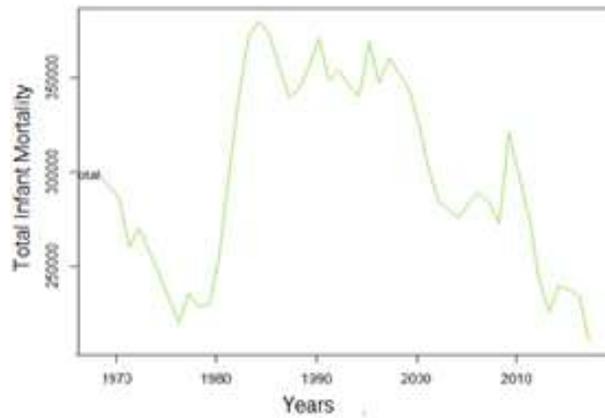


Fig. 1: Rainbow Plot of Total Infant Mortality of Pakistan during 1968-2017

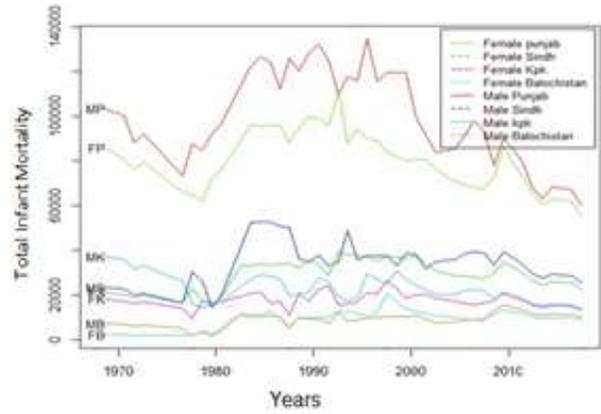


Fig. 2: Rainbow Plot of Total Infant Mortality of Pakistan during 1968-2017 Province and Gender wise

Table 3: Goodness of Fit of ARIMA Model using Bottom-Up, Top-Down and Optimal Combination Techniques

| Province    | Gender | MPE       |          |                     | MAPE      |          |                     | RMSE      |           |                     |
|-------------|--------|-----------|----------|---------------------|-----------|----------|---------------------|-----------|-----------|---------------------|
|             |        | Bottom Up | Top Down | Optimal Combination | Bottom Up | Top Down | Optimal Combination | Bottom Up | Top Down  | Optimal Combination |
| Punjab      | Male   | -1.41     | -17.18   | -1.03               | 6.47      | 10.38    | 5.80                | 8160.92   | 6018.2020 | 7348.60             |
|             | Female | -1.19     | -1.50    | -1.00               | 5.31      | 9.06     | 4.74                | 5161.29   | 4780.17   | 3905.38             |
| Sindh       | Male   | -1.78     | -1.95    | -1.60               | 13.18     | 20.00    | 13.26               | 6398.34   | 7372.21   | 6377.55             |
|             | Female | -0.65     | -3.36    | -0.35               | 8.28      | 15.41    | 7.97                | 3531.57   | 3106.95   | 3459.39             |
| KPK         | Male   | -2.81     | -1.52    | -2.94               | 10.76     | 9.34     | 10.85               | 3468.73   | 6189.99   | 3453.44             |
|             | Female | -3.01     | -3.35    | -3.06               | 11.69     | 8.65     | 11.15               | 3093.40   | 3356.94   | 3088.42             |
| Balochistan | Male   | -5.54     | -4.32    | -5.86               | 21.39     | 20.44    | 21.06               | 2050.74   | 3209.79   | 2026.49             |
|             | Female | -0.58     | 0.20     | -1.36               | 17.18     | 14.62    | 17.32               | 2238.75   | 2602.76   | 2224.30             |

Table 4: Comparison of Total Infant Mortality Forecasts using Bottom-Up, Top-Down and Optimal Combination Techniques during 2018-2027

| Years               | 2018     | 2019     | 2020     | 2021     | 2022     | 2023     | 2024     | 2025     | 2026     | 2027     |
|---------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| Bottom-Up           | 213869.0 | 213649.2 | 213176.0 | 212156.8 | 203933.8 | 203900.7 | 203782.1 | 203535.4 | 203043.6 | 202189.1 |
| Top-Down            | 227033.3 | 216799.4 | 196960.9 | 198449.6 | 197992.6 | 196088.1 | 195295.3 | 194211.4 | 193297.5 | 191190.7 |
| Optimal Combination | 221121.9 | 221292.7 | 211372.0 | 197663.5 | 197598.5 | 197624.2 | 197722.4 | 198533.3 | 196195.8 | 196153.9 |

period 2018-27 using Bottom-Up, Top-Down and Optimal Combination techniques taking ARIMA as base. The Hierarchical time series techniques illustrated the moderate decline in infant Mortality total during this period.

**Rainbow and Predicted Rainbow plots using Hierarchical Time Series**

Figure 1 displays the pattern of total infant mortality of Pakistan during the years 1968-2017. Initially, in 1968, the total infant mortality was around 0.29 million and then decreased gradually till 1980 with minor falsification around the year’s 1971 and 1978-79. The rainbow plot showed a drastic change in total infant mortality during the years 1982-83 which was about 3.7 million, then declined during the next one, two years, and then again whirled around 3.5 million till 1998. The pattern also exhibits the decreasing trend considerably till 2017 with some fluctuations during the years about 2007-10 as well as 2015 whereas total infant mortality in 2017 is on the minimum scale during the last 50 years. Figure 2 presents the rainbow plot of male

female total Infant Mortality (TIM) for each province along with gender wise during the years 1968-2017. The pattern manifests that TIM was highest in Punjab than the other provinces, in addition, the male TIM also higher than female TIM. On the other hand, TIM was lowest in Balochistan, it might be because of lesser population size. Figure indicates that the TIM decreased during the decade 1970-80 whereas increased drastically during the next two decades 1980-2000 with slight distortions and noticeable variation for Punjab and Sindh. The TIM spined around the count 5000-29000 with some ups and downs after 1990 whereas TIM decreased radically after 2000’s. NIPS (2017-2018) illustrated that the factors affect the total infant mortality in Pakistan i.e., education of mother, birth spacing and child size at birth whereas Patel, 2020 described that the Immune deficiency, respiratory illness, congenital malformations, low birth weight and smaller birth size with complications are the leading causes of infant deaths in Pakistan. Figures 3, 4 and 5 display the predicted rainbow pattern of male-female

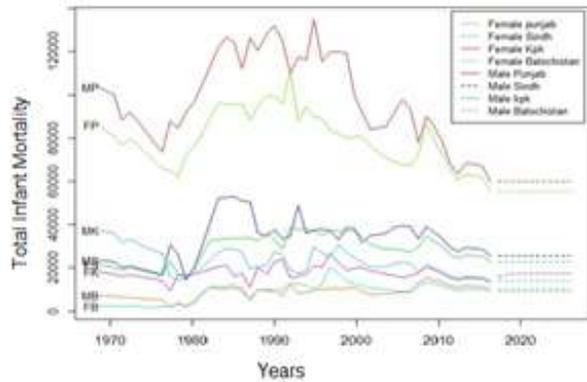


Fig. 3: Predicted Rainbow Plot of Total Infant Mortality of Province and Gender wise using Bottom-up Technique

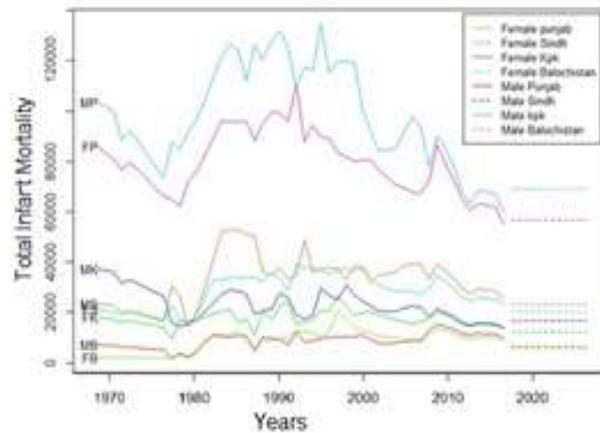


Fig. 4: Predicted Rainbow Plot of Total Infant Mortality of Province and Gender wise using Top-down Technique

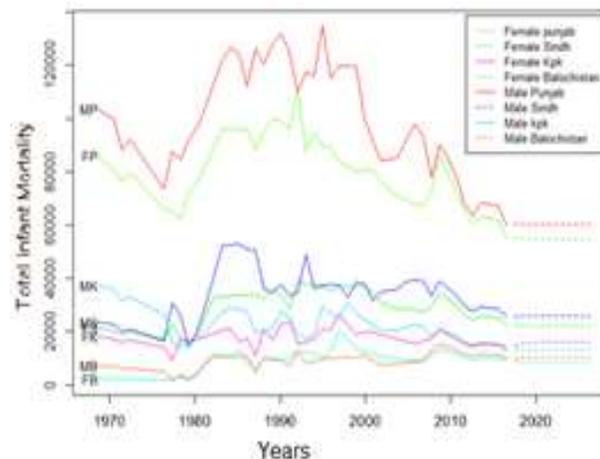


Fig. 5: Predicted Rainbow Plot of Total Infant Mortality of Province and Gender wise using Optimal Combination Technique

total infant mortality as well as province wise using the Bottom-Up, Top-Down and Optimal Combination techniques respectively. The spikes are little bit higher for male infants than the female infants, it might be because of the population sex ratio. In addition, the male-female total infant mortality was also forecasted during the period 2018-27 for four provinces using the hierarchical time series techniques. All the projections are comparable and differ slightly from each other with that of findings of Table 4. However, due to the consistency of the Optimal Combination technique, its projections may be considered and preferred.

### Conclusion and Recommendations

The analysis of total infant mortality using the hierarchical time series techniques i.e., Optimal combination, Bottom-Up and Top-Down techniques using exponential and ARIMA as base and interpretation stated that the Optimal Combination hierarchical time series technique, taking base ARIMA, performed better than the Bottom-Up and Top-Down techniques. The goodness of fit of the fitted three hierarchical time series models taking ARIMA as a base was also checked using the accuracy measures i.e., MPE, MAPE and, RMSE. It has been concluded that the Optimal Combination time series fitted better than the Bottom-Up and Top-Down techniques. The projection for the next ten years was also computed. It is pathologically established that Infant mortality and life expectancy are inversely related to each other. Luckily, the total infant mortality is being decreased gradually, but still, there is a need to take thoughtful measures to reduce it drastically and to meet the minimum infant mortality targets. The study may be also executed at sub-national levels and the findings would be beneficial for the Government policy decisions regarding the allocation of current and future resources. In Addition, some persuasive characteristics on infant mortality total may also be included in an extensive study.

### Authors' Contribution

All authors contributed equally in preparing this manuscript. All authors read and approved the final draft before final publication.

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